Uncertainty as a principle

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(Received 14 March 2005; accepted 24 May 2010; published online 1 July 2010)

Abstract: Since the presentation of the laws of Newton, all scientific development has been based on determinism. However, with the rise of quantum mechanics, uncertainty entered on a microscopic level. For that reason complementarity was considered to be a necessity to describe reality, but no suitable description of uncertainty could be found. It is also possible, however, to tackle uncertainty from a mathematical perspective, which has been overlooked until now. To this end, a mathematical formalism is presented, based on the concept that determinate and indeterminate manifestations can be considered to be mutually independent, occurring joined in nature in such a manner that one of both dominates an observation. The constructed complementary language is subsequently applied to geometric space, time, and marking. As a first result, Maxwell's laws are derived in a rather simple way. In principle, the strong force, gravity, the gravitational lens, and dark matter can be identified in just one observational description. Thus by giving uncertainty its right place in the world, a powerful model is obtained that serves as a conceptual basis for a unification theory. © *2010 Physics Essays Publication*. [DOI: 10.4006/1.3453731]

Résumé: Depuis la présentation des lois de Newton, tout développement scientifique était basé sur le déterminisme. Pourtant, avec l'arrivée de la mécanique quantique, l'incertitude a fait son entrée à l'échelle microscopique. C'est pour cette raison que la complémentarité fut considérée comme nécessaire pour décrire la réalité, mais aucune description satisfaisante de l'incertitude n'avait encore été trouvée. Il est cependant possible de considérer l'incertitude d'un point de vue mathématique, ce qui n'a encore jamais été vu à ce jour. Un formalisme mathématique est présenté, basé sur l'idée que des manifestations déterminées et indéterminées peuvent être considérées comme indépendantes l'une de l'autre mais dans la nature liées de telle sorte que l'une des deux domine une observation. Le langage construit et complémentaire est ensuite appliqué à l'espace géométrique, au temps et au marquage. Comme premier résultat, les lois de Maxwell sont dérivées d'une façon plutôt simple. Dans une seule description d'observation, la force forte, la force de gravitation, la lentille gravitationnelle et la matière noire peuvent en principe être identifiées. Ainsi, en donnant à l'incertitude sa juste place dans le monde, nous obtenons la base conceptuelle d'une théorie d'unification sous la forme d'un puissant modèle.

Key words: Indeterminism; Quantum mechanics; Schrödinger's Cat; Measurement Problem; Maxwell's Equations; Gravity; Gravitational Lens; Unification Theory.

I. HISTORICAL BACKGROUND

Since the discovery of the laws of Newton, all science has been deterministic: scientific progress was understood to be the minimization of indeterminism. The discovery of the quantum of action by Planck in 1900 marks the natural boundary of classical physics. His assumption that the radiation energy of a body is transmitted in quanta, rather than continuously, inspired Einstein in 1905 to assume that light in general is a discontinuous phenomenon. In 1913, N. Bohr proposed that the quantum hypothesis was valid for light as well as for matter.[1] In 1924, de Broglie derived an equation predicting that all atomic particles can act like a train of light waves, and in 1927 it was experimentally shown that the diffraction pattern of electrons, when striking a nickel crystal, were analogous to known wave patterns, by which particle-wave duality was confirmed.[2] Schrödinger finished a wave-mechanical formalism of quantum mechanics in 1926, reaching the border of classical physics: at the moment of measurement its expressiveness ends, which is called the "collapse of the wave function."

In 1927, W. Heisenberg presented the uncertainty principle at an atomic level as an attempt to connect this formalism and observational experience. According to this principle it is impossible to determine the position and the velocity of a particle at the same time.[3] This was the introduction of uncertainty in physics. In the discussions that took place in 1927 between Heisenberg *et al.* and others, Bohr proposed to incorporate the particle-wave dualism in the theory from the very beginning. Heisenberg proposed the term "complementarity," and they agreed that the uncertainty relations had to be considered as a special case of a more general complementarity in nature. However, Bohr never succeeded in finding a description of complementarity or of uncertainty that was suitable for physics.[3,4] The concept of uncertainty was hard to grasp, because there was no mathematical description

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of the subject available at the time. A solution to the impasse was found by Bohr and Heisenberg: the deterministic certainty of Newton's mechanics was replaced by the artifact of probabilistic randomness from quantum mechanics, eliminating the requirement for something that could be imagined. It worked very well in practice, so this made it possible to continue physical research without solving the problem.

Gradually the conviction took form that uncertainty was a microscopic phenomenon that could ultimately be phased out in general physics. This soothing thought was shattered by Schrödinger in 1935.[5] He invented a surprising thought experiment with a cat, pent up with a radioactive device linked to a poisoning system, which he used to demonstrate that microscopic uncertainty can be magnified to macroscopic size: "It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy…" But because he himself did not believe this to be an accurate representation of reality, he added a way out of the conundrum at the end of the sentence: "…which can then be *resolved* (italics added by this author) by direct observation." He seems to consider the magnification of uncertainty to be a nonscientific boobytrap that must be resolved and hurries back to classical, sound ideas: "That prevents us from so naively accepting as valid a 'blurred model' for representing reality." In fact, there is nothing to resolve; the observation is simply that we experience uncertainty about the condition of the cat. We can only obtain a decisive answer by completing the experiment—by opening the box. On the other hand, he was quite aware of this and concludes with a splendid correction: "In itself it would not embody anything unclear or contradictory." Nevertheless the scientific world was not able to accept this invasion of uncertainty in daily life and named his didactic construction the "Cat Paradox" accordingly.

This seemed to be the starting shot for the search for scientific ways to banish uncertainty from physics and to search for a renewed type of deterministic physics. Uncertainty and indeterminism were generally considered to be concepts beyond imagination, so to free physics from this problem it was considered no longer necessary to construct theories that reflected daily life experiences, like the falling of an apple from a tree. Imagination was only used for naming the many elementary particles that were found with the help of probabilistic methods. The standard model was developed in 1970–1973, based on the discovery of quarks, leptons, and force carriers, but although extremely useful in practical research, it is far from complete: it does not include gravity, dark matter, or dark energy. String theory, in development since 1968 and undergoing its second superstring revolution since 1995, is a purely abstract theory. This vast and complex theory, consistent with the standard model, contains equations for which the exact form cannot be determined; these have to be approximated, and thus this theory suffers from indeterminism.

Despite all efforts to remove uncertainty, it has resurfaced time and time again. Returning to the time before Newton, Pascal (1623–1662) remarked, according to a posthumously published book: "Tout ce qui est incomprehensible, ne laisse pas d'être" (Things that cannot be understood, do not cease to exist).[6] A theory in which indeterminism is removed is the Many-Worlds Interpretation (MWI), under development since 1957. It is based on the idea that myriads of worlds exist in the universe, in addition to the world we are aware of. In 2002, Vaidman wrote: "The existence of the other worlds makes it possible to remove randomness and action at a distance from quantum theory and thus from all physics," and: "The MWI is a deterministic theory for a physical Universe and it explains why a world appears to be indeterministic for human observers."[7] Unfortunately this is incompatible with the foundations of physics, although it might be useful as a stepping stone. Physics is based on concepts and theories explaining the behavior of non-living nature as it *appears* to us by observation. There is no reason to accept determinate appearances and reject indeterminate ones, except from force of habit; if nature is able to appear indeterministic to human observers, it is the task of physicists to adapt their concepts and theories. By replacing human observers with nonhuman ones, the problem is only pushed forward to the future: observing is ultimately a human activity.

Since no one has been successful in truly eliminating uncertainty, it might be so important that it needs a place of its own. Instead of neglecting uncertainty or fighting against it as if it were the big enemy of physics, we should investigate it as a concept of full value. Instead of using nonscientific terms for it, we must construct mathematically correct terms. It is time to give uncertainty its right place in the world.

II. PRELIMINARY

We pick up the history by searching for language to describe complementarity in which uncertainty is interpreted as an independent concept, not just as a lack of certainty. There is a promising possibility to tackle the subject from a mathematical point of view that has been overlooked until now. In 1955, von Weizsäcker proposed to tackle uncertainty in a mathematical manner by conceiving complementarity from a logical perspective.[8] Moreover, in 1974, Jammer suggested a definition of complementarity interpretation:

> "A given theory admits a complementarity interpretation if the following conditions are satisfied:
> (a) It contains (at least) two descriptions A_1 and A_2 of its substance-matter;
> (b) A_1 and A_2 refer to the same universe of discourse;
> (c) Neither A_1 nor A_2, if taken alone, accounts exhaustively for all phenomena of this universe;
> (d) A_1 and A_2 are mutually exclusive in the sense that their combination into a single description would lead to logical contradictions."[3]

If, indeed, phenomena are intrinsically dualistic, they must be expressible as being based on a collection of qualities that appear to us in a complementary way. For this reason, we introduce two abstract concepts to describe an arbitrary quality, which on the one hand imply mutual exclusiveness but on the other hand refer to one and the same quality: "determinate manifestation" and "indeterminate manifestation." These are defined as a pair of complementary attributes of the chosen quality. Additionally, we assume that attributes can be more or less important in specific combinations, which influences the ultimate observation.

The mathematical implications of this framework are deduced in a rather straightforward way from two axioms and one exclusion principle. Subsequently this mathematical language is applied to known physical concepts such as space and time. The abstract concepts of determinism and indeterminism are represented in suitable observational spaces, leading to descriptions of possible observations. Interaction between two phenomena is derived in order to be able to introduce a measuring device. Finally, by comparing these premade descriptions of observations with well known experimental results, we check the extent to which nature can be described in this way.

A. The Heisenberg event

A Heisenberg event or H-event is an elementary spacetime event expressed in complementary terms. If we represent its determinate mathematical attributes as D and its indeterminate ones as U, the H-event can be described as the set $\{D,U\}$. The selected letter U represents "uncertain" as one of the possible circumscriptions of indeterminism.

An elementary spacetime event is (P,S,t) in which P is an n-dimensional point and S an n-dimensional, infinite space; the dimensional indices are suppressed. To create a complementary description from this, the attributes P, S, and t have to be modified to become complementary ones. The point P, describing ultimate geometric contraction, can be considered as a determinate attribute without any adaption. Space S describes ultimate expansion, but it includes P, and thus the requirement that they exclude each other is not met. This problem can be solved by excluding the point from the space, thus selecting $S\backslash P$ for the indeterminate geometrical attribute. Now both attributes are complementary, excluding each other and together covering S completely.

Considering time, the real figure t can be considered as a determinate attribute without any adaption. An indeterminate attribute for time is the only missing attribute, and this can be created by adding an imaginary time indication $i\times t$ (in which $i=\sqrt{-1}$) to the collection. Then t and $i\times t$ can be considered to be complementary, excluding each other and together covering the complex plane completely. Thus each of the two elements of the set $\{D,U\}$ can again be written as a set:

$$D=\{P,t\},$$

$$U=\{S\setminus P,i\times t\}, \tag{1}$$

so the H-event can be described by:

$$\{D,U\}=\{\{P,t\},\{S\setminus P,i\times t\}\}. \tag{2}$$

In general, any creature can be considered to be a collection of H-events.

The interaction between collections of H-events is defined as a phenomenon φ.

Notice that, as a consequence, an H-event is not a phenomenon; it can only manifest itself by interacting with another H-event. For that reason, the H-event can be considered as the potential to interact. A characteristic part of the manifestation of the phenomenon φ is defined as a quality κ_i (kappa), which is a tensor containing a selection of attributes of the interacting H-events.

A determinate attribute of quality κ_i is denoted as D_i, an indeterminate one as U^i. We assume that all we want to know about φ can be described by n qualities $(\kappa_1,\kappa_2,\ldots,\kappa_n)$. The representation of a quality κ_i in an observational space is defined as an observation.

Here we foresee a problem. If an H-event would manifest as a result of some interaction, its determinate attribute is D, and this would be represented in an observational space; the result would be a totally determinate observation. But according to the Heisenberg principle this is impossible: each experimental result implies an amount of uncertainty. Thus we have to introduce from the very beginning some way of combining determinate and indeterminate attributes, such that our description is guarded against totally determinate results. To meet this requirement, we postulate the first axiom:

Axiom 1: The attributes of qualities contribute to any observation in pairs, To combine two attributes x and y to form a pair, we introduce the joining operator $\bowtie$ (pronounced as "is joined with").

A joined pair $x\bowtie y$ *is defined as* x and y are necessarily observed together. It is mathematically obvious that if $x\bowtie y$ than $y\bowtie x$; moreover, if $x=y$ then $x\bowtie y$ is defined as x.

According to the axiom, the H-event has one joined pair, which is $D\bowtie U$. This is its only possible contribution to an observation, which seems not to be in agreement with the experimental results from quantum mechanics. Matter on an atomic scale can behave like a particle (moving along a predictable trajectory) as well as like a wave (movement subject to probability) and never as a combination of these two. This suggests, in contradiction with our axiom, that each of these two observations involves just one attribute (D or U) instead of a pair.

A way out of this problem can be found by assuming that there is indeed a pair involved, but one of its members is of minor importance. Experimental evidence for this assumption is the observation that the particlelike behavior never results in a perfectly determinate point on the photographic layer but rather in a slightly diffused one, which indicates some indeterminate influence. On the other hand, the wavelike behavior does not result in a completely scattered picture but rather in a pattern exhibiting ring-shaped spots devoid of particles, indicating some determinate influence. This is in agreement with the vision of von Weizsäcker in which both slight diffusion and ring shapes have to be considered as effects of secondary order.[9] To incorporate this difference in importance in our description, we postulate the second axiom.

Axiom 2: A joined pair of attributes contributes to the observation in such a way that one member is of major and the other of minor importance.

To provide the attributes with importance, we indicate the minor ones in lower case. Attributes of major importance are indicated by D and U and attributes of minor importance by d and u. In a joined pair, the major element will, by convention, be written first. In this way, the joined pair $D \bowtie U$ is divided into the two joined pairs $D \bowtie u$ and $d \bowtie U$. It is self-evident that one and the same attribute cannot be of major and minor importance at the same time, so the joined pairs $D \bowtie d$ and $U \bowtie u$ have no meaning.

The second axiom establishes no restrictions for the actual proportion of importance between the major and minor attributes of a pair; it only states that there is a difference in importance that is large enough to observe in a first approximation just one of them. This is in agreement with Heisenberg who likewise did not give any indication for the range of imprecision in his uncertainty principle. As the axiom also poses no restriction as to *which* attribute will be of major importance, this means both can occur in an observation: the determinate and the indeterminate one.

As a result of the second axiom, qualities have to be expressed in terms of major and minor attributes. Because these are originating in H-events, we not only have to assign these complementary attributes but also a major and a minor variant of each, so we have to extend the description of the H-event $\{D,U\}$ to $\{D,U,d,u\}$. Obviously minor attributes have no meaning if there is no interaction with another H-event.

B. Genes and chromosomes

Before describing interaction between two H-events, we must establish a few definitions so we can organize attributes in a practical manner.

A gene g is a joined pair of attributes, which are assigned importance. Thus a gene contains a major and a minor attribute of one and the same H-event or of two different H-events. In general, with P_i as a major attribute of H_i and q_j as a minor attribute of H_j:

$$g = (P_i \bowtie q_j). \tag{3}$$

A genetic set G is the collection of all genes belonging to one phenomenon.

The genetic set of one H-event is

$$G = \{g_1, g_2\} = \{(D \bowtie u), (U \bowtie d)\}. \tag{4}$$

To indicate the combined occurrence of two genes g_i and g_j in one observation, we introduce the following: The *link operator* $\propto$ (pronounced as "is linked to"). $g_i \propto g_j$ is defined as g_i and g_j occur combined in an observation. Obviously if $g_i \propto g_j$ then $g_j \propto g_i$; if $g_i = g_j$ then $g_i \propto g_j$ is defined as g_i. We assume that linking is not distributive over joining.

To indicate the impossibility of linking two genes g_i and g_j, we introduce the following: The *parallel operator* $\|$ (pronounced as "is parallel to"). $g_i \| g_j$ is defined as g_i and g_j cannot occur combined in an observation. Obviously, if $g_i \| g_j$ then $g_j \| g_i$.

A chromosome c is a specific property of the phenomenon, defined as a chain of linked genes.

A chromosome is a tensor. If $g_i = P \bowtie q$ and $g_j = R \bowtie s$ are two genes, then a chromosome is

$$c_{ij} = g_i \propto g_j = (P \bowtie q) \propto (R \bowtie s). \tag{5}$$

The set of chromosomes C contains all chromosomes of the phenomenon.

From quantum-mechanical experiments, we know that particle and wave behavior cannot be observed simultaneously in one experiment; apparently genes $(D \bowtie u)$ and $(U \bowtie d)$ of one and the same H-event cannot be linked. When genes of several H-events are linked to a chromosome, this can, in principle, also contain both a major determinate and a major indeterminate attribute of one and the same H-event H_i, and again D_i and U^i would be simultaneously observable. We exclude these types of genetic combinations by postulating:

The exclusion principle for genes says that a gene containing a determinate major attribute of H_i cannot link with a gene containing an indeterminate major attribute of the same H-event.

Thus, in general, the genes $(D_i \bowtie x)$ and $(U^i \bowtie y)$, in which x and y are minor attributes of the same or of another H-event, cannot link to a chromosome. This can be written as

$$(D_i \bowtie x) \| (U^i \bowtie y). \tag{6}$$

By reducing the influence of minor attributes x and y infinitely and assuming continuity, the exclusion principle can be written as

$$D_i \| U^i. \tag{7}$$

A complementary observation ω is defined as the representation of a chromosome in a suitable observational space. This will be indicated by placing the represented chromosome between square brackets. Thus the complementary observation of the above mentioned zero-order chromosome is

$$\omega(g_i) = [P \bowtie q] \tag{8}$$

and that of the first-order chromosome:

$$\omega(g_i \propto g_j) = [g_i \propto g_j] = [(P \bowtie q) \propto (R \bowtie s)]. \tag{9}$$

The complementary set Ω is defined as the representation of a set of chromosomes in an observational space; its elements are called complementary observations.

A classical observation is defined as the limit of a complementary observation if the influence of the minor attributes is infinitely reduced. The classical set O is the set of all classical observations.

The observational sets are the complementary set together with the classical set.

An appearance A_i is a set of two elements; the first is an element of the classical set and the second the corresponding element of the complementary set.

$$A_i = \{\{O_i, \Omega_i\}\}. \tag{10}$$

The set of appearances A contains all possible appearances of a phenomenon.

The four possible appearances of the phenomenon are collected in a set:

$$A=\{A_1,A_2,A_3,A_4\} =\{\{O_1,\Omega_1\},\{O_2,\Omega_2\},\{O_3,\Omega_3\},\{O_4,\Omega_4\}\}. \quad (11)$$

C. Interaction between H-events

Consider two H-events H_1 and H_2 described by $\{D_1,U^1,d_1,u^1\}$ and $\{D_2,U^2,d_2,u^2\}$ interacting with each other and producing phenomenon φ_{12}. Their genetic sets are

$$G_1(H_1)=\{(D_1\bowtie u^1),(U^1\bowtie d_1)\} \quad \text{with} \quad (D_1\bowtie u^1)\parallel(U^1\bowtie d_1),$$

$$G_2(H_2)=\{(D_2\bowtie u^2),(U^2\bowtie d_2)\} \quad \text{with} \quad (D_2\bowtie u^2)\parallel(U^2\bowtie d_2). \quad (12)$$

Indices of indeterminate attributes are written as superscripts of the symbol for the purpose of clarity. To find the set of chromosomes of φ_{12}, we have to construct suitable genes. There are two distinct possibilities: we link the four genes of H_1 and H_2 such that they remain unchanged, as $(D_1\bowtie u^1)\propto(D_2\bowtie u^2)$, or we link them such that their minor attributes are exchanged, as $(D_1\bowtie u^2)\propto(D_2\bowtie u^1)$. We suppose that both types are present in the set of chromosomes of φ_{12}. Because of the exclusion principle, not all combinations are allowed. An example of a forbidden link is $(D_1\bowtie u^1)\propto(U^1\bowtie u^2)$, because $(D_1\bowtie u^1)\parallel(U^1\bowtie u^2)$; an example of an allowed link is $(D_2\bowtie u^1)\parallel(U^1\bowtie u^2)$.

In this way we obtain the set of first-order chromosomes of φ_{12}, containing eight elements:

$$C_1(\varphi_{12})=\left\{\begin{matrix}(D_1\bowtie u^1)\propto(D_2\bowtie u^2), & (D_1\bowtie u^2)\propto(D_2\bowtie u^1)\\ (U^1\bowtie d_1)\propto(U^2\bowtie d_2), & (U^1\bowtie d_2)\propto(U^2\bowtie d_1)\\ (D_1\bowtie u^1)\propto(U^2\bowtie d_2), & (D_1\bowtie d_2)\propto(U^2\bowtie u^1)\\ (D_2\bowtie u^2)\propto(U^1\bowtie d_1), & (D_2\bowtie d_1)\propto(U^1\bowtie u^2)\end{matrix}\right\}. \quad (13)$$

These first-order chromosomes are again linked with each other, observing the exclusion principle, by which we obtain the set of second order chromosomes of φ_{12}, containing four elements:

$$C_2(\varphi_{12})=\left\{\begin{matrix}(D_1\bowtie u^1)\propto(D_2\bowtie u^2)\propto(D_1\bowtie u^2)\propto(D_2\bowtie u^1)\\ (U_1\bowtie d^1)\propto(U^2\bowtie d_2)\propto(U^1\bowtie d_2)\propto(U^2\bowtie d_1)\\ (D_1\bowtie u^1)\propto(U^2\bowtie d_2)\propto(D_1\bowtie d_2)\propto(U^2\bowtie u^1)\\ (D_2\bowtie u^2)\propto(U^1\bowtie d_1)\propto(D_2\bowtie d_1)\propto(U^1\bowtie u^2)\end{matrix}\right\}. \quad (14)$$

Notice that the genes of the first two elements originate from one H-event, while the other two have a mixed origin. Because of the exclusion principle, interaction chromosomes of higher order do not exist, so this set contains the longest possible chromosomes. In the following example, we consider only this set. By representing the chromosomes in an observational space, we obtain the complementary set, containing four elements:

$$\Omega(\varphi_{12})=\left\{\begin{matrix}[(D_1\bowtie u^1)\propto(D_2\bowtie u^2)\propto(D_1\bowtie u^2)\propto(D_2\bowtie u^1)]\\ [(U_1\bowtie d^1)\propto(U^2\bowtie d_2)\propto(U^1\bowtie d_2)\propto(U^2\bowtie d_1)]\\ [(D_1\bowtie u^1)\propto(U^2\bowtie d_2)\propto(D_1\bowtie d_2)\propto(U^2\bowtie u^1)]\\ [(D_2\bowtie u^2)\propto(U^1\bowtie d_1)\propto(D_2\bowtie d_1)\propto(U^1\bowtie u^2)]\end{matrix}\right\}. \quad (15)$$

The classical set is obtained by reducing the minor attributes to the limit:

$$O(\varphi_{12})=\{[D_1\propto D_2],[U^1\propto U^2],[D_1\propto U^2],[D_2\propto U^1]\}. \quad (16)$$

The four possible appearances of the phenomenon are collected in a set

$$A=\{\{O_1,\Omega_1\},\{O_2,\Omega_2\},\{O_3,\Omega_3\},\{O_4,\Omega_4\}\}. \quad (17)$$

Thus the set of appearances is

$$A(\varphi_{12})=\left\{\begin{matrix}\{[D_1\bowtie D_2],[(D_1\bowtie u^1)\propto(D_2\bowtie u^2)\propto(D_1\bowtie u^2)\propto(D_2\bowtie u^1)]\}\\ \{[U^1\bowtie U^2],[(U_1\bowtie d^1)\propto(U^2\bowtie d_2)\propto(U^1\bowtie d_2)\propto(U^2\bowtie d_1)]\}\\ \{[D_1\bowtie U^2],[(D_1\bowtie u^1)\propto(U^2\bowtie d_2)\propto(D_1\bowtie d_2)\propto(U^2\bowtie u^1)]\}\\ \{[D_2\bowtie U^1],[(D_2\bowtie u^2)\propto(U^1\bowtie d_1)\propto(D_2\bowtie d_1)\propto(U^1\bowtie u^2)]\}\end{matrix}\right\}. \quad (18)$$

III. THE FIRST QUALITY

As the first quality to investigate, κ_1, we choose "geometrical presence," so we need geometric attributes to describe an H-event. But because we know experimentally that these might change with time and because we are used to thinking in terms of spacetime, we extend this to "spacetime presence."

First, we choose the *major attributes* D and U. Nothing can be more contracted than a point; it is the ultimate contraction, so we choose $D=P$. Nothing can be more expansive than an infinite space, so we choose $U=S\backslash P$. We complete the major attributes with time attributes. We are used to considering time as a one-dimensional real parameter, but we have to design two complementary attributes. Fortunately mathematics gives us a tool for expressing a one-dimensional magnitude in complementary language—by introducing imaginary figures—even if we have no idea what the sense of an imaginary time could be. If we choose the determinate attribute to be a real number t and the indeterminate attribute an imaginary number $i\times t$ (with $i=\sqrt{-1}$), they exclude each other and together fill the entire complex plane, so they can be considered as being complementary.

Next we consider the *minor attributes* d and u. For the minor space u, we choose a less expansive, more defined space: the inner space of a sphere s, which is the set of an infinite amount of points with a distance less than r (with r real and finite) to an arbitrary point; we choose the point P. No restriction to the magnitude of the radius is needed; it may be of atomic or astronomic magnitude, or it may be on a human scale.

For the minor point d, we choose a less contracted point by spreading it out as a pellicle p over the surface of s. Because in real space we know only points, no thinned points, the representation of p in real space, which is $[p]$, is a real minor point P_m existing somewhere on pellicle p.

In this way, the minor geometric system $s\cup p$ is constructed out of the two complementary attributes s and p, excluding each other and together covering it completely.

Notice that the minor space s is not a subspace of S, because P is an element of s but not of S. Notice also that in the major geometric system the major point is located in the center, while in the minor geometric system the minor point (the pellicle) is located at the surface.

We complete the minor attributes with time attributes, chosen to be an infinitesimally small part of the major system, so the differentials dt and $d(t\times i)=i\times dt$.

Summarizing, the chosen attributes of space are

$$D=P, \quad U=S\setminus P, \quad d=p, \quad u=s. \tag{19}$$

The chosen attributes of time are

$$D=t, \quad U=i\times t, \quad d=dt, \quad u=i\times dt. \tag{20}$$

Then the spacetime presence of an H-event H_n can be described by

$$\{D_n,U^n,d_n,u^n\}=\{\{P_n,t\},\{S^n\setminus P_n,i\times t\},\{p_n,dt\},\{s^n,i\times dt\}\}. \tag{21}$$

For clarity, the indices of determinate attributes are written as subscripts and those of indeterminate attributes as superscripts. An H-event can be considered to be a generalized point in complementary spacetime. It describes where and when the event is, in major as well as minor terms. According to our classical feeling, the determined attributes P and p seem to reflect the "presence" and the indeterminate ones S and s seem to reflect the "absence," but in complementary terms both of them are involved when we talk about presence. Note that distance plays no role.

Therefore, the geometric genetic set of H-event H_n is [see Eq. (4)]:

$$G(H_n,\kappa_1)=\{\{P_n,t\}\bowtie\{s^n,i\times dt\},\{S^n\setminus P_n,i\times t\}\bowtie\{p_n,dt\}\}=\{\{P_n\bowtie s^n,t\bowtie i\times dt\},\{S^n\setminus P_n\bowtie p_n,i\times dt\}\}. \tag{22}$$

We insert the attributes of κ_1 of the two H-events into Eq. (15) and, for the sake of convenience, split the equations into a space part and a time part.

The complemental set for the space attributes is

$$\Omega(\varphi_{12})=\left\{\begin{array}{l}[(P_1\bowtie s^1)\propto(P_2\bowtie s^2)\propto(P_1\bowtie s^2)\propto(P_2\bowtie s^1)]\\ [(S^1\setminus P_1\bowtie p^1)\propto(S^2\setminus P_2\bowtie p_2)\propto(S^1\setminus P_1\bowtie p_2)\propto(S^2\setminus P_2\bowtie p_1)]\\ [(P_1\bowtie s^1)\propto(S^2\setminus P_2\bowtie p_2)\propto(P_1\bowtie p_2)\propto(S^2\setminus P_2\bowtie s^1)]\\ [(P_2\bowtie s^2)\propto(S^1\setminus P_1\bowtie p_1)\propto(P_2\bowtie p_1)\propto(S^1\setminus P_1\bowtie s^2)]\end{array}\right\}, \tag{23}$$

and the classical set is

$$O(\varphi_{12},\text{3-space})=\{[P_1\propto P_2],[S^1\setminus P_1\propto S^2\setminus P_2],[P_1\propto S^2\setminus P_2],[P_2\propto S^1\setminus P_1]\}. \tag{24}$$

We define joining of space attributes such that a major point P_i, existing inside minor space s^j (so $P_i\in s^j$), or upon pellicle p_j (so $P_i\in p_j$), joins with this minor attribute forming a union of their points. If P_i is not existing inside minor space s^j (so $P_i\notin s^j$), or upon pellicle p_j (so $P_i\notin p_j$), then the joining is an empty space. So

$$P_i \bowtie s^j = s^j(\text{if } P_i \in s^j), \quad P_i \bowtie s^j = \varnothing\,(\text{if } P_i \notin s^j),$$

$$P_i \bowtie p_j = p_j(\text{if } P_i \in p_j), \quad P_i \bowtie p_j = \varnothing\,(\text{if } P_i \notin p_j). \tag{25}$$

We define linking of space attributes as taking the intersection of geometrical attributes, so

$$A \propto b = A \cap b. \tag{26}$$

For the observational space, we choose three-dimensional space. We carry out the operations as far as possible and assume that the order in which geometrical joining, linking, and representation in three-dimensional space are performed is of no consequence. Dimensional indices are suppressed.

The observational space sets for two interacting H-events are

$$\Omega(\varphi_{12},\text{3-space}) = \left\{\begin{matrix} [s^1 \cap s^2 \cap (P_1 \bowtie s^2) \cap (P_2 \bowtie s^1)] \\ [p_1 \cap p_2] \\ [s^1 \cap p_2 \cap (P_1 \bowtie p_2)] \\ [s^2 \cap p_1 \cap (P_2 \bowtie p_1)] \end{matrix}\right\},$$

$$O(\varphi_{12},\text{3-space}) = \{[P_1 \cap P_2],[S^1 \setminus P_1 \cap S^2 \setminus P_2],[P_1 \cap S^2 \setminus P_2],[P_2 \cap S^1 \setminus P_1]\}. \tag{27}$$

The set of appearances is the combination of all available information about the phenomenon that can be observed. Its first element is a set of two elements: the first classical element and the first complementary element; they can be observed simultaneously. Thus the set of appearances contains four elements. The sequence of the two observational elements Ω_i and O_i is changed because in experimental observation the classical element O_i will be noticed first. To observe the influence of minor attributes, more equipment is usually required. In this way, combining all corresponding elements, we obtain the following.

The set of space appearances, generated by two interacting H-events, is

$$A(\varphi_{12},\text{3-space}) = \left\{\begin{matrix} \{P_1 \cap P_2, s^1 \cap s^2 \cap (P_1 \bowtie s^2) \cap (P_2 \bowtie s^1)\} \\ \{S^1 \setminus P_1 \cap S^2 \setminus P_2, [p_1 \cap p_2]\} \\ \{P_1 \cap S^2 \setminus P_2, [s^1 \cap p_2 \cap (P_1 \bowtie p_2)]\} \\ \{P_2 \cap S^1 \setminus P_1, [s^2 \cap p_1 \cap (P_2 \bowtie p_1)]\} \end{matrix}\right\}$$

$$(\text{with:} P_i \bowtie s^j = s^j(\text{if } P_i \in s^j),\ P_i \bowtie s^j = \varnothing\,(\text{if } P_i \notin s^j),\ P_i \bowtie p_j = p_j(\text{if } P_i \in p_j),\ P_i \bowtie p_j = \varnothing\,(\text{if } P_i \notin p_j)). \tag{28}$$

For two interacting identical H-events, the observational space sets reduce to

$$\Omega(\varphi_{12},\text{3-space}) = \{[(P \bowtie s)],[(S \setminus P \bowtie p)]\},$$

$$O(\varphi_{12},\text{3-space}, H_1 = H_2) = \{[P],[S \setminus P]\} = \{P, S \setminus P\}. \tag{29}$$

Assuming that the time is running identically for both H-events so $t_1 = t_2$, the observational sets for the time attributes are

$$\Omega(\varphi_{12},\text{time}) = \left\{\begin{matrix} [(t \bowtie i \times dt) \propto (t \bowtie i \times dt) \propto (t \bowtie i \times dt) \propto (t \bowtie i \times dt)] \\ [(i \times t \bowtie dt) \propto (i \times t \bowtie dt) \propto (i \times t \bowtie dt) \propto (i \times t \bowtie dt)] \\ [(t \bowtie i \times dt) \propto (i \times t \bowtie dt) \propto (t \bowtie dt) \propto (i \times t \bowtie i \times dt)] \\ [(t \bowtie i \times dt) \propto (i \times t \bowtie dt) \propto (t \bowtie dt) \propto (i \times t \bowtie i \times dt)] \end{matrix}\right\},$$

$$O(\varphi_{12},\text{time}) = \{[t \propto t],[i \times t \propto i \times t],[t \propto i \times t],[t \propto i \times t]\}. \tag{30}$$

We define joining of time attributes as their addition. We define linking of time attributes as multiplication to introduce the second order of time into the description, so $x \propto y = x + y$. Thus

$$t \propto dt = t \times dt = 1/2 \times dt^2 \tag{31}$$

and because this is a qualitative discussion, the factor $\frac{1}{2}$ will be suppressed.

Then we carry out the operations. The complementary time set, generated by two interacting H-events, is

$$\Omega(\varphi_{12},\text{time}) = \begin{Bmatrix} [(t+i\times dt)] \\ [(i\times t+dt)] \\ [(t+i\times dt)\propto(i\times t+dt)\propto(t+dt)\propto(i\times t+i\times dt)] \\ [(t+i\times dt)\propto(i\times t+dt)\propto(t+dt)\propto(i\times t+i\times dt)] \end{Bmatrix}. \quad (32)$$

The representation in a one-dimensional real space of the sum of two complex numbers is equal to the representation of the sum of the separate representations, so for the third and fourth elements:

$$[(t+i\times dt)\propto(i\times t+dt)\propto(t+dt)\propto(i\times t+i\times dt)] = [t+i\times dt]\propto[i\times t+dt]\propto[t+dt]\propto[i\times t + i\times dt] = t\propto dt\propto(t+dt). \quad (33)$$

Carrying out the linking, assuming $dt \ll t\times dt$ and thus approximating $t+dt$ with t, the third and fourth elements reduce to

$$t\propto dt\propto t = t\propto dt = t\times dt = dt^2. \quad (34)$$

Then, by taking the real parts of the first and second elements as well, we obtain

$$\Omega(\varphi_{12},\text{time}) = \{t,dt,dt^2,dt^2\}. \quad (35)$$

To obtain the classical set, we take the limit of reduced influence of the minor appearances $i\times dt$ and dt in the complementary set (32), by which this reduces to the classical set:

$$O(\varphi_{12},\text{time}) = \{[t],[i\times t],[t\times i\times t],[t\times i\times t]\} \quad (36)$$

By representing in real time, the classical time set, generated by two interacting H-events, is

$$O(\varphi_{12},\text{time}) = \{t,0,t^2,t^2\}. \quad (37)$$

Combining all corresponding elements, we obtain the following: The set of time appearances, generated by two interacting H-events, is

$$A(\varphi_{12},\text{time}) = \{\{t,t\},\{0,dt\},\{t^2,dt^2\},\{t^2,dt^2\}\}. \quad (38)$$

We combine space and time parts (28) and (38) to four-dimensional spacetime appearances. In the following expression, we suppress the indication of the phenomenon φ_{12} to focus our attention entirely on the considered quality κ_1. Recalling the set of appearances in general [see Eq. (11)]:

$$A = \{\{O_1,\Omega_1\},\{O_2,\Omega_2\},\{O_3,\Omega_3\},\{O_4,\Omega_4\}\}. \quad (39)$$

A phenomenon, generated by two interacting Heisenberg events H_1 and H_2, with

$$H_n{:}\{D_n,U^n,d_n,u^n\} = \{\{P_n,t\},\{S^n\setminus P_n,i\times t\},\{p_n,dt\},\{s^n,i\times dt\}\} \quad (40)$$

is described by the set of spacetime appearances:

$$A(\kappa_1) = \begin{Bmatrix} \{\{P_1\cap P_2,t\},\{s^1\cap s^2\cap(P_1\bowtie s^2)\cap(P_2\bowtie s^1),t\}\} \\ \{\{S^1\setminus P_1\cap S^2\setminus P_2,0\},\{[p_1\cap p_2],dt\}\} \\ \{\{P_1\cap S^2\setminus P_2,t^2\},\{[s^1\cap p_2\cap(P_1\bowtie p_2)],dt^2\}\} \\ \{\{P_2\cap S^1\setminus P_1,t^2\},\{[s^2\cap p_1\cap(P_2\bowtie p_1)],dt^2\}\} \end{Bmatrix}$$

$$\text{with } P_i\bowtie s^j = s^j(\text{if } P_i\in s^j),\quad P_i\bowtie s^j=\varnothing\,(\text{if } P_i\notin s^j),\quad P_i\bowtie p_j = p_j(\text{if } P_i\in p_j),\quad P_i\bowtie p_j=\varnothing\,(\text{if } P_i\notin p_j). \quad (41)$$

A. Identification of the first quality

In the following discussion, the set of appearances is considered for eight distinct relative geometric positions of the two H-events with respect to each other. We begin by considering spacetime presence by describing the set of appearances for equal radii and coinciding major points. We then continue with a selection of different relative positions of the major points (equal radii). Lastly, we consider two cases with different radii. The distance between the major points is denoted as P_{12}, and the axis is denoted as P_1P_2.

Although the four elements of the set of appearances describe one and the same relative position of the H-events, mind that they are not observed simultaneously; the set is the collection of possible manifestations occurring in *distinct observations*. Each element of the set contains classical observational information, based on major attributes (major spaces and major points), enriched with complementary observational information, based on minor attributes (closed spaces and pellicles); they are, by definition, compatible. In fact, the classical and complementary descriptions in one element occur *simultaneously*, with the second completing the first as a kind of a fine structure.

We consider, to the extent possible, stable interaction between the two H-events, so in general we do not allow one case to develop into another one. If, for instance, the geometric conditions define a major point on a pellicle, this point is not supposed to leave this pellicle. Our selection is far from complete; in reality any interaction will develop such that a distinct case emerges into another one and many more H-events will be involved, resulting in an overwhelming number of possibilities.

Indicated geometric attributes are only observable as far as the corresponding element supplies the necessary time attribute; movements in the complementary element are restricted to the indicated minor space.

In Ω_1 the three-dimensional object $s^1 \cap s^2$ is called a *discus*. In O_2 the codomain of the major spaces $S^1 \backslash P_1 \cap S^2 \backslash P_2$, describing the open space outside of P_1 and P_2, will be abbreviated as $S^1 \backslash \{P_1, P_2\}$.

In Ω_2, the representation of the pellicle intersection in real three-dimensional space $[p_1 \cap p_2]$, is a real minor point P_m existing somewhere on the circle $p_1 \cap p_2$; this will be denoted as

$$[p_1 \cap p_2] = P_m(p_1 \cap p_2). \tag{42}$$

It is combined with a first order of time, so it moves with a constant velocity along the one-dimensional, circular path $p_1 \cap p_2$. This point is assumed to represent both pellicles p_1 and p_2; thus it will be considered as two coinciding pellicle points. If P_m turns clockwise, as seen from major point P_1, then it turns counterclockwise, as seen from major point P_2.

If the pellicles completely coincide, the representation in real three-dimensional space $[p]$ reduces to two pellicle points, coinciding on p in one real minor point P_m, so

$$[p_1 \cap p_2] = [p] = P_m(p). \tag{43}$$

If, on the contrary, the pellicles have only one point X_m in common, this is considered as the intersection of two circles with radius zero. The representation X_m is considered as the limit of the two pellicle points, still rotating according to the first derivative of time, so

$$[p_1 \cap p_2] = X_m(p_1 \cap p_2). \tag{44}$$

X_m is called a *turning point* of the pellicles.

We interpret the zero time in A_2 as the capacity to describe discontinuous change, thus acting as a singular element of the set. If element A_1, A_3, or A_4 of the same set describes an observation such that in the course of time a singular point is required, this can only be described in A_2.

In Ω_3 the representation of the space-pellicle intersection for $P_1 \in p_2$ is equal to $[s^1 \cap p_2]$; the curved surface $s^1 \cap p_2$ is called a *dot*; the real minor point P_m exists somewhere on this dot, which is written as

$$[s^1 \cap p_2 \cap (P_1 \bowtie p_2)] = P_m(s^1 \cap p_2). \tag{45}$$

If $P_1 \in p_2$ and the minor space encloses the pellicle, then the real minor point P_m exists somewhere on the total pellicle:

$$[s^1 \cap p_2 \cap (P_1 \bowtie p_2)] = [s^1 \cap p_2] = [p_2] = P_m(p_2). \tag{46}$$

In A_4 the representation is mirrored with respect to A_4.

It is advisable to use four transparent disks in several sizes (one small, one large, and two of an identical, intermediate size) with border and central point marked; this will facilitate comprehension of the following substantially.

B. Eight cases of two H-events, interacting in spacetime

Case 1: $P_1 = P_2$, $r_1 = r_2$. The H-events are identical in spacetime. According to Eq. (41),

$$A(\kappa_1) = \begin{Bmatrix} \{\{P,t\},\{s,t\}\} \\ \{\{S \setminus P,0\},\{P_m(p),dt\}\} \\ \{\{\varnothing,t^2\},\{\varnothing,dt^2\}\} \\ \{\{\varnothing,t^2\},\{\varnothing,dt^2\}\} \end{Bmatrix}. \tag{47}$$

A_1 classically, in O_1, describes a major point P at a point of time t; complementary to this, in Ω_1, two coinciding minor spaces are described. This can be identified as a *real particle*, having an extensiveness s; we interpret the shared minor space s as related to *inert mass*.

A_2 classically, in O_2, describes the space outside the major point, in which the time is zero; complementary to this, a minor point P_m (p) on p is described with a first derivation of time, in other words, a point moving over a curved surface with a constant velocity. Because the impression is that of a particle, having no content and thus massless, existing in the open space, we identify this as a *virtual particle*.

A_3 and A_4 describe nothing, so interacting identical H-events are not accelerating. In summary, at an atomic scale the set can be identified as *free particles*, both real and virtual.

Case 2: $0 < P_{12} < r$. Both major points exist inside the codomain. According to Eq. (41),

$$A(\kappa_1) = \begin{Bmatrix} \{\{P,t\},\{s^1 \cap s^2,t\}\}, \\ \{\{S \setminus \{P_1,P_2\},0\},\{P_m(p_1 \cap p_2),dt\}\} \\ \{\{P_1,t^2\},\{\varnothing,dt^2\}\} \\ \{\{P_2,t^2\},\{\varnothing,dt^2\}\} \end{Bmatrix}. \tag{48}$$

A_1 describes in Ω_1 the minor codomain $s^1 \cap s^2$, a discus that is observable because both major points are inside of it. This can be identified as two particles in a nucleus, bound by the *strong force*. The shared minor codomain is considered to be related to *inert mass*. Notice that it is only complementarily observable, not classically.

A_2 describes in O_2 a space with zero time and in Ω_2 a minor point P_m on the intersection of two pellicles $p_1 \cap p_2$, which is a circle in a plane perpendicular to the axis P_1P_2; it is the border of the discus $s^1 \cap s^2$. According to the time-attribute dt, the minor point P_m moves linearly over this circle, so it moves in one of the two possible directions with a constant velocity; the circle is called a *minor track*.

A_3 and A_4 each describe an accelerated point mass, but if they were to move toward each other, their pellicles would shrink to zero; then the major points would coincide with the pellicles, which is not possible in the established geometrical conditions. Moving apart from each other is also impossible because the major points exist inside the static discus $s^1 \cap s^2$. Thus they are not accelerating, and the major points are not observable in A_3 and A_4, so the set reduces to

$$A(\kappa_1) = \left\{ \begin{array}{l} \{\{\emptyset,t\},\{s^1 \cap s^2,t\}\} \\ \{\{S \setminus \{P_1,P_2\},0\},\{P_m(p_1 \cap p_2),dt\}\} \\ \{\{\emptyset,t^2\},\{\emptyset,dt^2\}\} \\ \{\{\emptyset,t^2\},\{\emptyset,dt^2\}\} \end{array} \right\}. \quad (49)$$

The set can be used to explain the results of quantum-mechanical experiments, showing the duality of matter. The minor appearance of the discus in A_1 can be identified as the spot on the photographic layer, produced by an H-event after crossing a vacuum, manifesting its *particle character*. The minor track, which can be observed in A_2, can be identified as the ring where the H-event can hit the screen, manifesting its *wave character*. The pellicle intersection is an extremely thin presence, so in general it will be necessary to add effects over the course of time to obtain an observation. For the description of concentric rings, resulting from interaction with other H-events in a regular crystal, we need to develop a set of appearances with more than two H-events, which is beyond our scope. These interferencelike patterns are described by the Schrödinger equations, but the so-called "collapse of the wave function" at the moment the photographic plate is reached remained a mysterious detail. However, combined with the classical observation, this is not a problem because of the zero-time attribute in A_2: if time stands still, an abrupt change is possible.

If this set would still be valid for a large number of H-events, at an astronomic level a huge number of major points can be collected inside a minor codomain. Their pellicles, relatively close to each other if the major points are relatively close to each other, could in an extreme case add to a band of pellicles; between the band and the collection of major points, a free part of their shared minor spaces would exist. All together this would be a *very heavy object*, with a huge amount of minor points traveling through a band around this object, clockwise as well as counterclockwise.

Case 3: $r < P_{12} < 2 \times r$. There is a minor codomain. The major points are outside of it, and not on the pellicle. According to Eq. (41),

$$A(\kappa_1) = \left\{ \begin{array}{l} \{\{\emptyset,t\},\{\emptyset,t\}\} \\ \{\{S \setminus \{P_1,P_2\},0\},\{P_m(p_1 \cap p_2),dt\}\} \\ \{\{P_1,t^2\},\{\emptyset,dt^2\}\} \\ \{\{P_2,t^2\},\{\emptyset,dt^2\}\} \end{array} \right\}. \quad (50)$$

This set is identical to the one in case 2, but the geometric conditions are different.

A_1: *empty geometries*. There is a nonempty minor codomain, so there is inert mass, but it is not observable.

A_2: Ω_2 describes a circular *minor track* over which a minor point $P_m(p_1 \cap p_2)$ is moving with a constant velocity.

A_3 and A_4: *accelerated major points*. If they move away from each other, they would shift apart because the major points are not attached to the pellicles and emerge into case 5 (touching pellicles). If they are approaching, the major points

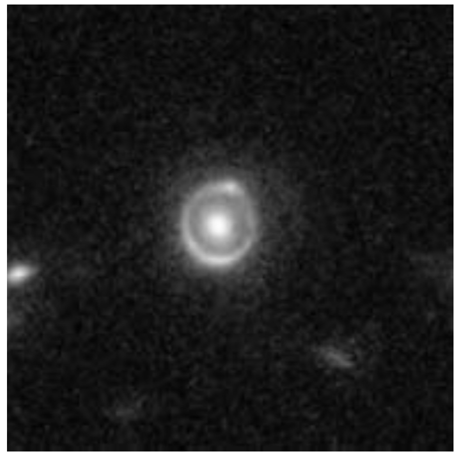

FIG. 1. (Color online) Picture 0038+4133 by NASA (Ref. 10). Two objects, the first a very heavy one, are in line with the telescope. The ring is considered as the intersection of their pellicles.

meet each others' pellicles, so case 3 would emerge. The case is only stable if the H-events are not accelerating; then the major points P_1 and P_2 are not observable in A_3 and A_4. The stable set can be reduced to

$$A(\kappa_1) = \left\{ \begin{array}{l} \{\{\emptyset,t\},\{\emptyset,t\}\} \\ \{\{S \setminus \{P_1,P_2\},0\},\{P_m(p_1 \cap p_2),dt\}\} \\ \{\{\emptyset,t^2\},\{\emptyset,dt^2\}\} \\ \{\{\emptyset,t^2\},\{\emptyset,dt^2\}\} \end{array} \right\}. \quad (51)$$

At an atomic level, the set can describe H-events drifting apart with a constant velocity, which can be identified as particles existing in a *gas*. Apparently the pellicles adapt to this type of movement; a slight cohesion is produced by the minor point $P_m(p_1 \cap p_2)$. If they are approaching each other and meet at the turning point, the directions reverse and they start to move in opposite directions.

Cases 2 and 3 combined. If on an astronomical level two objects—where one object is very heavy, see case 2—are in a similar relative position as described in case 3, and if the telescope is in line with both objects, a *full ring* $[p_1 \cap p_2]$ can be observed. This ring is actually composed of an extremely large number of minor points $P_m(p_1 \cap p_2)$ in the band of pellicles of the very heavy object, combined with the band of pellicles of the second celestial object. At the points of intersection of the minor tracks, the minor points have an opportunity to collide. If we assume that colliding pointlike objects produce light, this light can travel to earth and if the intensity is high enough, this is visible as a ring around the two objects (one hidden behind the other). This effect has indeed been captured by the Hubble space telescope: a full ring can be seen (Fig. 1).

The very heavy object lies in front of the lighter one and causes a phenomenon known as a *gravitational lens*, because it is ascribed according to the traditional concept of gravity. According to our model, however, the beam of light travels through the minor space of the heavy object. The picture shows a slight *redshift*, indicating a drifting of the ring away from the observer; this type of movement is part of the description in case 3. If the intersection of the two bands of pellicles, one being of the heavy object, is indeed observed, the same effect can be expected if the light body is in front of the heavy one; then, in a reversed situation, we would expect a slight blueshift. If the very heavy object is really an anisotropic cluster of objects, *nodal points* with a higher intensity exist in the intersection and we expect to see nodes in the ring. This is shown in the picture mentioned earlier: two nodes can be distinguished, almost facing each other.

Case 4: $r_1=r_2=r$ and $P_{12}=r$. The major points are attached to each others' pellicles. According to Eq. (41),

$$A(\kappa_1)=\begin{Bmatrix}\{\{\varnothing,t\},\{\varnothing,t\}\}\\ \{\{S\setminus\{P_1,P_2\},0\},\{P_m(p_1\cap p_2),dt\}\}\\ \{\{P_1,t^2\},\{P_m(s^1\cap p_2),dt^2\}\}\\ \{\{P_2,t^2\},\{P_m(s^2\cap p_1),dt^2\}\}\end{Bmatrix}. \tag{52}$$

A_1: *empty geometries*. The spherical codomain is not observable because the second major point is not present in the codomain; so although inert mass is present, it is not observable in this element.

A_2: Minor point $P_m(p_1\cap p_2)$ travels with a constant velocity over the minor track $p_1\cap p_2$, which is a one-dimensional circular path.

Ω_3: The minor point $P_m(s^1\cap p_2)$ is accelerating over the dot $s^1\cap p_2$, which is a half-spherical curved surface with P_1 in its center. Because its space is restricted to $s^1\cap p_2$, the minor point circles around the axis P_{12} in one of the two possible directions. Because of compatibility, we assume that this is the same minor point as described in Ω_2, and thus the circular path is the border of the dot. Thus,

$$P_m(s^1\cap p_2)=P_m(p_1\cap p_2). \tag{53}$$

Ω_4: the same as Ω_3 in symmetrical sense, thus

$$P_m(s^2\cap p_1)=P_m(p_1\cap p_2). \tag{54}$$

This implies that the minor point moves over the dot border in a plane halfway between P_1 and P_2.

O_3: major point P_1 simultaneously moves accelerating toward or away from P_2 (P_1 cannot circle around in the dot because it is the center). If P_1 moves toward P_2, its pellicle is taken with it, so this shrinks until the H-events meet each other with zero radii in the turning point (which can be described, because Ω_2 is not empty). This can be identified as the *weight* of H_1 with respect to H_2. Notice that, although the inert mass is not observable in A_1, its weight can be observed in A_3. The approaching major point is accompanied by the minor point $P_m(s^1\cap p_2)$, spinning around P_1, P_2 as it travels over the border of the dot in a plane perpendicular to P_{12}. This minor point makes a converging circular movement as P_1 approaches P_2.

A_4: The same applies, in symmetrical sense.

Summarizing so far, if the major points move toward each other, the set describes gravity. The set can be written as

$$A(\kappa_1)=\begin{Bmatrix}\{\{\varnothing,t\},\{\varnothing,t\}\}\\ \{\{S\setminus\{P_1,P_2\},0\},\{P_m(p_1\cap p_2),dt\}\}\\ \{\{P_1,t^2\},\{P_m(s^1\cap p_2),dt^2\}\}\\ \{\{P_2,t^2\},\{P_m(s^2\cap p_1),dt^2\}\}\end{Bmatrix}. \tag{55}$$

After some time, the major and minor points coincide in the turning point

$$P_m(p_1\cap p_2)=X_m(p_1\cap p_2). \tag{56}$$

There the H-events reflect: P_1 and P_2 continue to describe an accelerated movement, but in the opposite direction; this is possible because the direction is not fixed in the description. After reflection, the radii of the pellicles start to grow again. If this phenomenon occurs in a similar way for other H-events, it could be identified as a *naked singularity*. There is no limit for the accelerating divergence; the pellicles will stretch out accordingly, because the major points are attached to them. Again, the accelerating major point P_1 will be accompanied by a minor point, which after some time moves over the border of a dot of astronomical size. Summarizing, if the major points move away from each other, the set describes dark matter.

Case 5: $P_{12}=2\times r$. The pellicles touch each other in minor point X_m. According to Eq. (41),

$$A(\kappa_1)=\begin{Bmatrix}\{\{\varnothing,t\},\{\varnothing,t\}\}\\ \{\{S\setminus\{P_1,P_2\},0\},\{X_m(p_1\cap p_2),dt\}\}\\ \{\{P_1,t^2\},\{\varnothing,dt^2\}\}\\ \{\{P_2,t^2\},\{\varnothing,dt^2\}\}\end{Bmatrix}. \tag{57}$$

A_1 can be identified as a vacuum.

In A_2, because the time in the major space $S\backslash\{P_1,P_2\}$ (which is $S^1\backslash P_1\cap S^2\backslash P_2$) is zero, the two pellicle points represented in X can pop up as two coinciding major points P_3 and P_4. Then, because major points without attributes are by definition not allowed in the description, two new major spaces $S_3\backslash X$ and $S_4\backslash X$ must be created as well, to provide major indeterminate attributes. Simultaneously, four new minor attributes must be created: two minor spaces s_3 and s_4 with pellicles p_3 and p_4; they have radius zero to be unobservable in Ω_2. Thus appearance A_2 can be identified as the creation of a pair of H-events.

A_3 and A_4 describe accelerating point masses. A circular movement is not possible because no suitable geometry is described, so they are approaching or diverging. Because the major points are not attached to the pellicles, the geometric conditions will only be satisfied for the one moment, at a point of time t, that the pellicles are touching each other. Thus all attention in this case is for A_2; as soon as A_3 or A_4 is observed, the time runs and the case ends. In summary, the set describes the creation of a pair of H-events in vacuum.

Case 6: $P_{12}>2\times r$. No minor attributes are shared. The two H-events share only their major spaces $S^1\backslash P_1$ and $S^2\backslash P_2$. According to Eq. (41),

$$A(\kappa_1) = \begin{Bmatrix} \{\{\varnothing,t\},\{\varnothing,t\}\} \\ \{\{S\setminus\{P_1,P_2\},0\},\{\varnothing,dt\}\} \\ \{\{P_1,t^2\},\{\varnothing,dt^2\}\} \\ \{\{P_2,t^2\},\{\varnothing,dt^2\}\} \end{Bmatrix}. \quad (58)$$

Because the codomain is empty, A_1 can be identified as a vacuum.

A_2 describes an empty space outside of P_1 and P_2. A_3 and A_4 describe accelerating point masses. The most stable description of the set is obtained when the H-events are moving away from each other; then the set describes the expanding universe.

Cases 5 and 6 combined: If the major points in case 6 are approaching each other, the pellicles will touch each other after some time and case 5 emerges: a pair of H-events can be created. The new H-events can again interact to create new H-events and the resulting H-events can also mutually interact, so in principle an explosive multiplication of H-events is possible; a complete universe can be created out of two H-events. This can be identified as the big bang.

Case 7: $P_{12}=r_1$ and $r_1 \gg r_2$. P_2 is attached to the large pellicle p_1. According to Eq. (41),

$$A(\kappa_1) = \begin{Bmatrix} \{\{\varnothing,t\},\{\varnothing,t\}\} \\ \{\{S\setminus\{P_1,P_2\},0\},\{P_m(p_1 \cap p_2),dt\}\} \\ \{\{P_1,t^2\},\{\varnothing,dt^2\}\} \\ \{\{P_2,t^2\},\{P_m(s^2 \cap p_1),dt^2\}\} \end{Bmatrix}. \quad (59)$$

A_1: *empty geometries*. There is a nonempty minor codomain, so there is inert mass, but it is not observable.

A_2: A one-dimensional, circular minor track with minor point $P_m(p_1 \cap p_2)$ moving over it at a constant velocity.

A_3: Accelerating motion of P_1 without a minor geometric attribute.

A_4: In Ω_4 a minor point $P_m(s^2 \cap p_1)$ exists on the dot $s^2 \cap p_1$. Because of compatibility, we assume that this is the same minor point as described in A_2, moving over the border of the dot, around the axis P_{12}:

$$P_m(s^2 \cap p_1) = P_m(p_1 \cap p_2). \quad (60)$$

In O_4, the major point P_2 is accelerating; this cannot be a circular movement over the dot, because it is the center. It cannot move to or from P_1 because P_1 is not attached to p_2 and thus another case would emerge. The remaining possibility is that, because P_2 is attached to the pellicle p_1, the complete dot is accelerating over p_1, simultaneously turning around the axis P_{12}. Then, because of symmetry, the acceleration of P_1 according to A_3 must be a rotation around P_2. But because in A_3 no geometric attribute is observable, this movement of P_1 is not observable and, again because of symmetry, the other major point is also unobservable; the major points can be replaced by empty geometries and the set reduces to

$$A(\kappa_1) = \begin{Bmatrix} \{\{\varnothing,t\},\{\varnothing,t\}\} \\ \{\{S\setminus\{P_1,P_2\},0\},\{P_m(p_1 \cap p_2),dt\}\} \\ \{\{\varnothing,t^2\},\{\varnothing,dt^2\}\} \\ \{\{\varnothing,t^2\},\{P_m(s^2 \cap p_1),dt^2\}\} \end{Bmatrix}. \quad (61)$$

This set can be identified as the geometric basis of an electron in a shell around a nucleus, including a spin movement. Notice that it is only complementarily observable; classically, nothing is observable.

Case 8: $P_{12}=r_2$ and $r_2 \ll r_1$. The second pellicle is inside the first and P_1 is attached to it, so $P_1 \in p_2$. According to Eq. (41),

$$A(\kappa_1) = \begin{Bmatrix} \{\{\varnothing,t\},\{\varnothing,t\}\} \\ \{\{S\setminus\{P_1,P_2\},0\},\{\varnothing,dt\}\} \\ \{\{P_1,t^2\},\{P_m(p_2),dt^2\}\} \\ \{\{P_2,t^2\},\{\varnothing,dt^2\}\} \end{Bmatrix}. \quad (62)$$

A_1: *empty geometries*. There is a small spherical minor codomain, so there is inert mass, but it is not observable because the second major point is not present inside.

A_2: An empty space outside of P_1 and P_2 in which the time is zero.

A_3: Ω_3 describes an accelerated minor point P_m (p_2) on pellicle p_2. According to O_3, major point P_1 accelerates simultaneously and because it is attached to p_2 we assume that it coincides with the representation of the pellicle, so

$$P_1 = P_m(p_2) \quad (63)$$

A_4: No minor geometric object, so analogous to case 7 the major points are not observable and the set reduces to

$$A(\kappa_1) = \begin{Bmatrix} \{\{\varnothing,t\},\{\varnothing,t\}\} \\ \{\{S\setminus\{P_1,P_2\},0\},\{\varnothing,dt\}\} \\ \{\{\varnothing,t^2\},\{P_m(p_2),dt^2\}\} \\ \{\{\varnothing,t^2\},\{\varnothing,dt^2\}\} \end{Bmatrix}. \quad (64)$$

In summary, this set can be identified as a virtual particle (see case 1). Like in case 7, the virtual particle is only complementarily observable; classically, nothing is observable.

Summarizing all cases, the sets of appearances of a phenomenon, generated by two inter acting H-events, can be identified as a variety of manifestations of mass, both stable and unstable, observable and unobservable. Thus quality κ_1, describing spacetime presence, can be identified as mass.

IV. THE SECOND QUALITY

By using geometrical attributes and representing two interacting H-events in four-dimensional spacetime, we have found answers for the question: "where and when is something happening?" Next we would like to be able to distinguish between two H-events, because then we can carry out measurements, knowing which H-event is the measuring device and which is the investigated one. So our next question is: "which one is it?" and to answer this we need to mark H-events. Thus, as the second quality to investigate, κ_2, we choose "marking."

By choosing the marking attributes such that they are compatible with the chosen spacetime attributes, we can combine the results later, so we design our marking attributes such that they can easily be applied to P, $S\backslash P$, p, and s. We derive the observational marking sets in an abstract way, considering the attributes only as mathematical objects. A suitable mathematical tool to mark an H-event is the vector field. For the determinate vector field, we choose an infinite, real, n-dimensional, radial oriented, time-dependent vector field $\vec{\mathbf{E}}(t)$ having a source of strength Q at a point P, which may be chosen arbitrarily. A vector of $\vec{\mathbf{E}}(t)$, indicated by $\vec{\mathbf{e}}(t)$, is called positive if it points from P to infinity. According to the first law of Gauss, the vectorial flux emerging from a closed surface in a vector field is proportional to the magnitude of the sources within this surface; applying the divergence theorem to this law yields the requirement for the chosen radial vector field $\vec{\mathbf{E}}(t)$:

$$\vec{\nabla} \cdot \vec{\mathbf{E}}(t) = Q \tag{65}$$

when integrating over a surface enclosing P. In this way the magnitudes of the vectors are determined, so a determinate vector $\vec{\mathbf{e}}$ is attached to each point of the event space S; thus $\vec{\mathbf{E}}(t)$ is a determinate vector field. There is not yet a vector defined in source P; to complete the vector field to all points in S, we define the zero-vector $\vec{\mathbf{o}}$ in P. This vector can be considered to be determinate, because its length is exactly zero. However, the zero vector has no marking features. In order to mark P in a determinate way, we attach the real number Q to it.

Then the H-event is major determinate marked by the set $\{Q, \vec{\mathbf{E}}(t)\}$. For the indeterminate vector field, we choose an infinite, real, n-dimensional, circular oriented, time-dependent vector field $\vec{\mathbf{B}}(t)$ with central point P. We also define a vector of $\vec{\mathbf{B}}(t)$, which will be indicated by $\vec{\mathbf{b}}(t)$. In each point of S, the direction of $\vec{\mathbf{b}}(t)$ is tangent to a spherical surface. In that case there is still an infinite number of possible directions for $\vec{\mathbf{b}}(t)$; even adjacent vectors might point in a variety of directions. Moreover, there is no information about the absolute value of $\vec{\mathbf{b}}(t)$. This means that these vectors have an infinite variety of absolute values and directions, although by definition there is only one vector in each point. In this way, an indeterminate vector $\vec{\mathbf{b}}(t)$ is attached to each point of S; thus $\vec{\mathbf{B}}(t)$ is an indeterminate vector field. Because of the definitions of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$, the inner product of $\vec{\mathbf{e}}(t)$ and $\vec{\mathbf{b}}(t)$ in each point of S is zero, so

$$\vec{\mathbf{E}}(t) \cdot \vec{\mathbf{B}}(t) = \vec{\mathbf{O}} \tag{66}$$

and because $\vec{\mathbf{E}}(t)$ is radial:

$$\vec{\nabla} \cdot \vec{\mathbf{B}}(t) = 0. \tag{67}$$

To complete the vector field $\vec{\mathbf{B}}(t)$ to all points in S, we define in P the zero vector $\vec{\mathbf{o}}$; this vector can be considered as indeterminate because its direction is totally indeterminate. To mark P in an indeterminate way, we attach the imaginary number $Q \times i$ (with $i = \sqrt{-1}$) to it. Thus P acts as an imaginary source of $\vec{\mathbf{B}}(t)$. Then the H-event is major indeterminate marked by the set $\{Q \times i, \vec{\mathbf{B}}(t)\}$.

The minor attributes have to be such that they have a connection with both major marks $\{Q, \vec{\mathbf{E}}(t)\}$ and $\{Q \times i, \vec{\mathbf{B}}(t)\}$, and that the resulting genes still have a vectorial character. We choose two operators, one geometrical: $d = \vec{\nabla} = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$ and one timelike: $u - \partial/\partial t$. Thus the H-event is minor determinate marked by the operator $\vec{\nabla}$ and minor indeterminate by the operator $\partial/\partial t$. We would like to introduce vector field variations in the descriptions, so we define the joining operator $\bowtie$ such that for a vector field $\vec{\mathbf{A}}$:

$$\vec{\mathbf{A}} \bowtie \partial/\partial t = \partial \vec{\mathbf{A}}/\partial t, \tag{68}$$

$$\vec{\mathbf{A}} \bowtie \vec{\nabla} = \vec{\nabla} \times \vec{\mathbf{A}}. \tag{69}$$

Summarizing an H-event is marked by the attributes

$$D = \{Q, \vec{\mathbf{E}}(t)\}$$

$$U = \{Q \times i, \vec{\mathbf{B}}(t)\} \tag{70}$$

on the conditions

$$\vec{\nabla} \cdot \vec{\mathbf{E}}(t) = Q \quad \text{and} \quad \vec{\nabla} \cdot \vec{\mathbf{B}} = 0,$$

$$d = \vec{\nabla},$$

$$u = \partial/\partial t.$$

The time dependency of the vector fields will be suppressed in the following notations.

So H_n can be described by

$$\{D_n, U^n, d_n, u^n\} = \{\{Q, \vec{\mathbf{E}}(t)\}, \{Q \times i, \vec{\mathbf{B}}(t)\}, \vec{\nabla}, \partial/\partial t\}. \tag{71}$$

Then the marking genetic set of H_n is

$$G(H_n, \kappa_2) = \{\partial \vec{\mathbf{E}}_n/\partial t, \vec{\nabla} \times \vec{\mathbf{B}}_n\}$$

on the conditions

$$\vec{\nabla} \cdot \vec{\mathbf{E}}_n(t) = Q_n \quad \text{and} \quad \vec{\nabla} \cdot \vec{\mathbf{B}}_n = 0. \tag{72}$$

The observational sets of a noninteracting H-event, considering marking, are

$$\Omega(\kappa_3) = \{\partial \vec{\mathbf{E}}_H/\partial t, \vec{\nabla} \times \vec{\mathbf{B}}_H\}$$

$$O(\kappa_3) = \{\{Q, \vec{\mathbf{E}}_H\}, \{0, \vec{\mathbf{B}}_H\}\} \tag{73}$$

on the conditions

$$\vec{\nabla} \cdot \vec{\mathbf{E}}(t) = Q \quad \text{and} \quad \vec{\nabla} \cdot \vec{\mathbf{B}} = 0.$$

Linking of marking numbers is defined as addition and linking of vector fields as vector addition.

We assume $dt_1 = dt_2 = dt$ and $\vec{\nabla}_1 = \vec{\nabla}_2 = \vec{\nabla}$. Then we insert the attributes of κ_2 into Eqs. (15) and (16).

The complemental time set for two interacting H-events is

$$\Omega(\kappa_2)=\left\{\begin{matrix}[\partial\vec{\mathbf{E}}_1/\partial t \propto \partial\vec{\mathbf{E}}_2/\partial t \propto \partial\vec{\mathbf{E}}_1/\partial t \propto \partial\vec{\mathbf{E}}_2/\partial t]\\ [\vec{\nabla}\times\vec{\mathbf{B}}_1 \propto \vec{\nabla}\times\vec{\mathbf{B}}_2 \propto \vec{\nabla}\times\vec{\mathbf{B}}_1 \propto \vec{\nabla}\times\vec{\mathbf{B}}_2]\\ [\partial\vec{\mathbf{E}}_1/\partial t \propto \vec{\nabla}\times\vec{\mathbf{B}}_2 \propto \vec{\nabla}\times\vec{\mathbf{E}}_1 \propto \partial\vec{\mathbf{B}}_2/\partial t]\\ [\partial\vec{\mathbf{E}}_2/\partial t \propto \vec{\nabla}\times\vec{\mathbf{B}}_1 \propto \vec{\nabla}\times\vec{\mathbf{E}}_2 \propto \partial\vec{\mathbf{B}}_1/\partial t]\end{matrix}\right\}, \tag{74}$$

which can be reduced to

$$\Omega(\kappa_2)=\left\{\begin{matrix}[\partial\vec{\mathbf{E}}_1/\partial t \propto \partial\vec{\mathbf{E}}_2/\partial t]\\ [\vec{\nabla}\times\vec{\mathbf{B}}_1 \propto \vec{\nabla}\times\vec{\mathbf{B}}_2]\\ [\partial\vec{\mathbf{E}}_1/\partial t \propto \vec{\nabla}\times\vec{\mathbf{B}}_2 \propto \vec{\nabla}\times\vec{\mathbf{E}}_1 \propto \partial\vec{\mathbf{B}}_2/\partial t]\\ [\partial\vec{\mathbf{E}}_2/\partial t \propto \vec{\nabla}\times\vec{\mathbf{B}}_1 \propto \vec{\nabla}\times\vec{\mathbf{E}}_2 \propto \partial\vec{\mathbf{B}}_1/\partial t]\end{matrix}\right\}. \tag{75}$$

The classical time set for two interacting H-events is

$$O(\kappa_2)=\left\{\begin{matrix}\{[Q_1 \propto Q_2],[\vec{\mathbf{E}}_1 \propto \vec{\mathbf{E}}_2]\}\\ \{[Q_1\times i \propto Q_2\times i],[\vec{\mathbf{B}}_1 \propto \vec{\mathbf{B}}_2]\}\\ \{[Q_1 \propto Q_2\times i],[\vec{\mathbf{E}}_1 \propto \vec{\mathbf{B}}_2]\}\\ \{[Q_2 \propto Q_1\times i],[\vec{\mathbf{E}}_2 \propto \vec{\mathbf{B}}_1]\}\end{matrix}\right\}. \tag{76}$$

The sets can be simplified by using a self-evident restriction: we require that, if the marked H-events are identical, the observational marking sets reduce to the representations of the genetic marking set of a noninteracting H-event. First, we consider the complementary set and then the classical set; the attributes of time are put aside in this discussion.

By dropping the indices, we obtain the complementary marking set for two identical H-events:

$$\Omega(\kappa_2)=\{[\partial\vec{\mathbf{E}}/\partial t],[\vec{\nabla}\times\vec{\mathbf{B}} \propto \nabla\times\vec{\mathbf{B}}],[\partial\vec{\mathbf{E}}/\partial t \propto \vec{\nabla}\times\vec{\mathbf{B}} \propto \vec{\nabla}\times\vec{\mathbf{E}} \propto \partial\vec{\mathbf{B}}/\partial t]\}, \tag{77}$$

and compare this with the complementary marking set of a noninteracting H-event (73):

$$\Omega(0,\kappa_2)=\{[\partial\vec{\mathbf{E}}_H/\partial t],[\vec{\nabla}\times\vec{\mathbf{B}}_H]\}. \tag{78}$$

The first elements are equal if $[\partial\vec{\mathbf{E}}/\partial t]=[\partial\vec{\mathbf{E}}_H/\partial t]$, which is valid if $[\vec{\mathbf{E}}]=[\vec{\mathbf{E}}_H]$.

The second elements are equal if $[\vec{\nabla}\times\vec{\mathbf{B}}]=[\vec{\nabla}\times\vec{\mathbf{B}}_H]$, which is valid if $[\vec{\mathbf{B}}]=[\vec{\mathbf{B}}_H]$.

The third element, $[\partial\vec{\mathbf{E}}/\partial t \propto \vec{\nabla}\times\vec{\mathbf{B}} \propto \vec{\nabla}\times\vec{\mathbf{E}} \propto \partial\vec{\mathbf{B}}/\partial t]$, being a chain of four genes of which the first two can be recognized as single genes, disappears if it, as a whole, is equal to one of the single elements $[\partial\vec{\mathbf{E}}_H/\partial t]$ or $[\vec{\nabla}\times\vec{\mathbf{B}}_H]$. If we require the first two to be equal, so

$$\partial\vec{\mathbf{E}}/\partial t=\vec{\nabla}\times\vec{\mathbf{B}}, \tag{79}$$

then

$$\partial\vec{\mathbf{E}}/\partial t \propto \vec{\nabla}\times\vec{\mathbf{B}}=\partial\vec{\mathbf{E}}/\partial t, \tag{80}$$

and the third element reduces to $\partial\vec{\mathbf{E}}/\partial t \propto \vec{\nabla}\times\vec{\mathbf{E}} \propto \partial\vec{\mathbf{B}}/\partial t$.

We can get rid of the last link of this expression by carrying out the linking:

$$\vec{\nabla}\times\vec{\mathbf{E}} \propto \partial\vec{\mathbf{B}}/\partial t=\vec{\nabla}\times\vec{\mathbf{E}}+\partial\vec{\mathbf{B}}/\partial t, \tag{81}$$

and requiring that the last two parts cancel each other:

$$\partial\vec{\mathbf{B}}/\partial t=-\vec{\nabla}\times\vec{\mathbf{E}} \tag{82}$$

by which the element reduces to

$$\partial\vec{\mathbf{E}}/\partial t \propto \vec{\nabla}\times\vec{\mathbf{E}} \propto \partial\vec{\mathbf{B}}/\partial t=\partial\vec{\mathbf{E}}/\partial t \tag{83}$$

and the third interaction element indeed equals the first element of the noninteracting H-event.

Next we consider the *classical* marking set for two identical H-events:

$$O(\kappa_2)=\{[\vec{\mathbf{E}} \propto \vec{\mathbf{E}}],[\vec{\mathbf{B}} \propto \vec{\mathbf{B}}],[\vec{\mathbf{E}} \propto \vec{\mathbf{B}}],[\vec{\mathbf{E}} \propto \vec{\mathbf{B}}]\} \\ =\{[\vec{\mathbf{E}}],[\vec{\mathbf{B}}],[\vec{\mathbf{E}} \propto \vec{\mathbf{B}}]\} \tag{84}$$

and compare this with the classical marking set (73) of a noninteracting H-event:

$$O(0,\kappa_2)=\{[\vec{\mathbf{E}}_H],[\vec{\mathbf{B}}_H]\}. \tag{85}$$

The first and second elements are equal if $[\vec{\mathbf{E}}]$ is chosen to be equal to $[\vec{\mathbf{E}}_H]$ and $[\vec{\mathbf{B}}]$ as equal to $[\vec{\mathbf{B}}_H]$. The third element disappears if this is the same as the first or the second, so if

$$[\vec{\mathbf{E}} \propto \vec{\mathbf{B}}]=[\vec{\mathbf{E}}_H] \quad \text{or} \quad [\vec{\mathbf{E}} \propto \vec{\mathbf{B}}]=[\vec{\mathbf{B}}_H]. \tag{86}$$

Thus we have to require

$$[\vec{\mathbf{E}} \propto \vec{\mathbf{B}}]=[\vec{\mathbf{E}}] \quad \text{or} \quad [\vec{\mathbf{E}} \propto \vec{\mathbf{B}}]=[\vec{\mathbf{B}}]. \tag{87}$$

This would be valid if $\vec{\mathbf{E}}=\vec{\mathbf{B}}$, but this is not possible because $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are by definition complementary. Carrying out the linking, $[\vec{\mathbf{E}}+\vec{\mathbf{B}}]$ has to be equal to $[\vec{\mathbf{E}}]$ or $[\vec{\mathbf{B}}]$, which is valid for $[\vec{\mathbf{B}}]=\vec{\mathbf{O}}$ or $\vec{\mathbf{E}}=\vec{\mathbf{O}}$, which means that one of the two vector fields has to be nonobservable. If $\vec{\mathbf{E}}=\vec{\mathbf{O}}$, then $\vec{\nabla}\times\vec{\mathbf{B}}=\vec{\mathbf{O}}$ and $\partial\vec{\mathbf{B}}/\partial t=\vec{\mathbf{O}}$, so this option represents an unmarked event. If $[\vec{\mathbf{B}}]=\vec{\mathbf{O}}$, then $\partial\vec{\mathbf{E}}/\partial t=\vec{\mathbf{O}}$, so in that case we are only restricted to a statically determined vector field $\vec{\mathbf{E}}$, with an indeterminate vector field $\vec{\mathbf{B}}$, which is not observable in a single H-event. This is a reasonable option.

Summarizing, the conditions for the marking vector fields have to be extended with

$$\partial\vec{\mathbf{E}}/\partial t=\vec{\nabla}\times\vec{\mathbf{B}}$$

$$\partial\vec{\mathbf{B}}/\partial t=-\vec{\nabla}\times\vec{\mathbf{E}} \tag{88}$$

and the chosen vector field attributes of an H-event are restricted to vector fields for which

$$\partial\vec{\mathbf{E}}/\partial t=\vec{\mathbf{O}} \quad \text{and} \quad [\vec{\mathbf{B}}]=\vec{\mathbf{O}} \tag{89}$$

is valid. We incorporate these restrictions in the marking sets. In the classical one, the second element is the linking of two indeterminate single vector fields, $\vec{\mathbf{B}}_1 \propto \vec{\mathbf{B}}_2$ of which is required that each is not independently observable. We assume that the link between them as also unobservable, as a result of which this element reduces to the zero field.

Then the complemental marking set of two interacting H-events is

$$\Omega(\kappa_2)=\begin{Bmatrix}[\partial\vec{\mathbf{E}}_1/\partial t\propto\partial\vec{\mathbf{E}}_2/\partial t]\\ [\vec{\nabla}\times\vec{\mathbf{B}}_1\propto\vec{\nabla}\times\vec{\mathbf{B}}_2]\\ [\partial\vec{\mathbf{E}}_1/\partial t\propto\vec{\nabla}\times\vec{\mathbf{B}}_2\propto\vec{\nabla}\times\vec{\mathbf{E}}_1\propto\partial\vec{\mathbf{B}}_2/\partial t]\\ [\partial\vec{\mathbf{E}}_2/\partial t\propto\vec{\nabla}\times\vec{\mathbf{B}}_1\propto\vec{\nabla}\times\vec{\mathbf{E}}_2\propto\partial\vec{\mathbf{B}}_1/\partial t]\end{Bmatrix}, \tag{90}$$

the classical one is

$$O(\kappa_2)=\begin{Bmatrix}\{[Q_1\propto Q_2],[\vec{\mathbf{E}}_1\propto\vec{\mathbf{E}}_2]\}\\ \{[Q_1\times i\propto Q_2\times i],\vec{\mathbf{O}}\}\\ \{[Q_1\propto Q_2\times i],\{\vec{\mathbf{E}}_1\propto\vec{\mathbf{B}}_2\}\}\\ \{[Q_2\propto Q_1\times i],\{\vec{\mathbf{E}}_2\propto\vec{\mathbf{B}}_1\}\}\end{Bmatrix}, \tag{91}$$

and the set of appearances, changing the sequence of Ω_i and O_i, is

$$A(\kappa_2)=\begin{Bmatrix}\{\{[Q_1\propto Q_2],[\vec{\mathbf{E}}_1\propto\vec{\mathbf{E}}_2]\},[\partial(\vec{\mathbf{E}}_1\propto\vec{\mathbf{E}}_2)/\partial t]\}\\ \{\{[Q_1\times i\propto Q_2\times i],\vec{\mathbf{O}}\},[\vec{\nabla}\times\vec{\mathbf{B}}_1\propto\vec{\nabla}\times\vec{\mathbf{B}}_2]\}\\ \{\{[Q_1\propto Q_2\times i],[\vec{\mathbf{E}}_1\propto\vec{\mathbf{B}}_2]\},[\partial\vec{\mathbf{E}}_1/\partial t\propto\vec{\nabla}\times\vec{\mathbf{B}}_2\propto\vec{\nabla}\times\vec{\mathbf{E}}_1\propto\partial\vec{\mathbf{B}}_2/\partial t]\}\\ \{\{[Q_2\propto Q_1\times i],[\vec{\mathbf{E}}_2\propto\vec{\mathbf{B}}_1]\},[\partial\vec{\mathbf{E}}_2/\partial t\propto\vec{\nabla}\times\vec{\mathbf{B}}_1\propto\vec{\nabla}\times\vec{\mathbf{E}}_2\propto\partial\vec{\mathbf{B}}_1/\partial t]\}\end{Bmatrix} \tag{92}$$

on the conditions

$$\vec{\nabla}\cdot\vec{\mathbf{E}}_n=Q_n,\quad \vec{\nabla}\cdot\vec{\mathbf{B}}_n=0,\quad \partial\vec{\mathbf{E}}_n/\partial t=\vec{\nabla}\times\vec{\mathbf{B}}_n,\quad \partial\vec{\mathbf{B}}_n/\partial t=-\vec{\nabla}\times\vec{\mathbf{E}}_n.$$

For *identically marked H*-events, thus where $Q_1=Q_2=Q$, each link reduces to one gene and the set reduces to

$$A(\kappa_2)=\begin{Bmatrix}\{\{[Q],[\vec{\mathbf{E}}]\},[\partial\vec{\mathbf{E}}/\partial t]\}\\ \{\{[Q\times i],\vec{\mathbf{O}}\},[\vec{\nabla}\times\vec{\mathbf{B}}]\}\\ \{\{[Q\propto Q\times i],[\vec{\mathbf{E}}\propto\vec{\mathbf{B}}]\},[\partial\vec{\mathbf{E}}/\partial t\propto\vec{\nabla}\times\vec{\mathbf{B}}\propto\vec{\nabla}\times\vec{\mathbf{E}}\propto\partial\vec{\mathbf{B}}/\partial t]\}\\ \{\{[Q\propto Q\times i],[\vec{\mathbf{E}}\propto\vec{\mathbf{B}}]\},[\partial\vec{\mathbf{E}}/\partial t\propto\vec{\nabla}\times\vec{\mathbf{B}}\propto\vec{\nabla}\times\vec{\mathbf{E}}\propto\partial\vec{\mathbf{B}}/\partial t]\}\end{Bmatrix} \tag{93}$$

on the conditions

$$\vec{\nabla}\cdot\vec{\mathbf{E}}_n=Q_n,\quad \vec{\nabla}\cdot\vec{\mathbf{B}}_n=0,\quad \partial\vec{\mathbf{E}}_n/\partial t=\vec{\nabla}\times\vec{\mathbf{B}}_n,$$
$$\partial\vec{\mathbf{B}}_n/\partial t=-\vec{\nabla}\times\vec{\mathbf{E}}_n.$$

Using these conditions,

$$\partial\vec{\mathbf{E}}/\partial t\propto\vec{\nabla}\times\vec{\mathbf{B}}\propto\vec{\nabla}\times\vec{\mathbf{E}}\propto\partial\vec{\mathbf{B}}/\partial t=\partial\vec{\mathbf{E}}/\partial t\propto\partial\vec{\mathbf{E}}/\partial t\propto -\partial\vec{\mathbf{B}}/\partial t\propto\partial\vec{\mathbf{B}}/\partial t=\partial\vec{\mathbf{E}}/\partial t, \tag{94}$$

remembering that a static $\vec{\mathbf{B}}$-field is not observable, thus

$$[\vec{\mathbf{E}}\propto\vec{\mathbf{B}}]=[\vec{\mathbf{E}}+\vec{\mathbf{B}}]=[\vec{\mathbf{E}}]+[\vec{\mathbf{B}}]=[\vec{\mathbf{E}}]+\vec{\mathbf{O}}=\vec{\mathbf{E}} \tag{95}$$

and representing it in a one-dimensional real space and a three-dimensional vector space, the set of marked appearances, generated by two identical marked H-events is

$$A(\kappa_2)=\begin{Bmatrix}\{\{Q,\vec{\mathbf{E}}\},\partial\vec{\mathbf{E}}/\partial t\}\\ \{\{0,\vec{\mathbf{O}}\},\vec{\nabla}\times\vec{\mathbf{B}}\}\\ \{\{Q,\vec{\mathbf{E}}\},\partial\vec{\mathbf{E}}/\partial t\}\\ \{\{Q,\vec{\mathbf{E}}\},\partial\vec{\mathbf{E}}/\partial t\}\end{Bmatrix} \tag{96}$$

on the conditions

$$\vec{\nabla}\cdot\vec{\mathbf{E}}_n=Q_n,\quad \vec{\nabla}\cdot\vec{\mathbf{B}}_n=0,\quad \partial\vec{\mathbf{E}}_n/\partial t=\vec{\nabla}\times\vec{\mathbf{B}}_n,$$
$$\partial\vec{\mathbf{B}}_n/\partial t=-\vec{\nabla}\times\vec{\mathbf{E}}_n.$$

For nonidentical marked H-events, to obtain the representation in an observational space we carry out the links (defined as addition), represent the added marking numbers in a one-dimensional real space and the n-dimensional vector fields in a three-dimensional vector space. The dimensions are suppressed in the notation. In the classical set, joining is carried out by addition. The marking numbers are represented in a real one-dimensional space and the fields in a real three-dimensional vector field. In the classical set, joining is carried out by addition. Remember that a static $\vec{\mathbf{B}}$-field is not observable, so

$$[\vec{\mathbf{E}}_1\propto\vec{\mathbf{B}}_2]=[\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2]=[\vec{\mathbf{E}}_1]+[\vec{\mathbf{B}}_2]=[\vec{\mathbf{E}}_1]+\vec{\mathbf{O}}=\vec{\mathbf{E}}_1. \tag{97}$$

The first complementary element can be written as

$$\partial\vec{\mathbf{E}}_1/\partial t+\partial\vec{\mathbf{E}}_2/\partial t=\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t \tag{98}$$

and the second as

$$\vec{\nabla}\times\vec{\mathbf{B}}_1+\vec{\nabla}\times\vec{\mathbf{B}}_2=\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2). \tag{99}$$

Similarly, the third and fourth can be rewritten. Then, with H_n generally described by

$$\{D_n, U^n, d_n, u^n\} = \{\{Q_n, \vec{\mathbf{E}}_n(t)\}, \{Q_n \times i, \vec{\mathbf{B}}_n(t)\}, \vec{\nabla}, \partial/\partial t\}, \tag{100}$$

the set of marked appearances, generated by two interacting H-events, is

$$A(\kappa_2) = \left\{ \begin{array}{l} \{\{\{Q_1 + Q_2, \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2\}\}, \{\partial(\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2)/\partial t\}\} \\ \{\{0, \vec{\mathbf{O}}\}, \{\vec{\nabla} \times (\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2)\}\} \\ \{\{\{Q_1, \vec{\mathbf{E}}_1\}\}, \{\partial(\vec{\mathbf{E}}_1 + \vec{\mathbf{B}}_2)/\partial t + \vec{\nabla} \times (\vec{\mathbf{E}}_1 + \vec{\mathbf{B}}_2)\}\} \\ \{\{\{Q_2, \vec{\mathbf{E}}_2\}\}, \{\partial(\vec{\mathbf{E}}_2 + \vec{\mathbf{B}}_1)/\partial t + \vec{\nabla} \times (\vec{\mathbf{E}}_2 + \vec{\mathbf{B}}_1)\}\} \end{array} \right\} \tag{101}$$

on the conditions

$$\vec{\nabla} \cdot \vec{\mathbf{E}}_n = Q_n, \quad \vec{\nabla} \cdot \vec{\mathbf{B}}_n = 0, \quad \partial \vec{\mathbf{E}}_n/\partial t = \vec{\nabla} \times \vec{\mathbf{B}}_n,$$
$$\partial \vec{\mathbf{B}}_n/\partial t = -\vec{\nabla} \times \vec{\mathbf{E}}_n.$$

Observing these results, the source Q can be identified as *electric charge* and the vector fields $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ as *electric and magnetic fields*, and the field conditions are similar to Maxwell's equations in vacuum. Thus *quality* κ_2, describing marking, can be identified as electromagnetic features. For practical reasons this can be shortened to *charge*.

Notice that a magnetic monopole cannot be observed: the source of the magnetic field is imaginary ($Q \times i$), so its representation in a real space is zero.

Now we combine the attributes of *marking* with those of *spacetime*. For clarity we add the attributes of time first and, after having considered the result, the attributes of space. Adding the set of time appearances to the set of marked appearances we obtain the following: The set of marked appearances combined with time, generated by two H-events, is

$$A(\kappa_2, \text{time}) = \left\{ \begin{array}{l} \{\{\{Q_1 + Q_2, \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2\}, t\}, \{\partial(\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2)/\partial t, t\}\} \\ \{\{\vec{\mathbf{O}}, 0\}, \{\vec{\nabla} \times (\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2), dt\}\} \\ \{\{\{Q_1, \vec{\mathbf{E}}_1\}, t^2\}, \{\partial(\vec{\mathbf{E}}_1 + \vec{\mathbf{B}}_2)/\partial t + \vec{\nabla} \times (\vec{\mathbf{E}}_1 + \vec{\mathbf{B}}_2), dt^2\}\} \\ \{\{\{Q_2, \vec{\mathbf{E}}_2\}, t^2\}, \{\partial(\vec{\mathbf{E}}_2 + \vec{\mathbf{B}}_1)/\partial t + \vec{\nabla} \times (\vec{\mathbf{E}}_2 + \vec{\mathbf{B}}_1), dt^2\}\} \end{array} \right\} \tag{102}$$

on the conditions

$$\vec{\nabla} \cdot \vec{\mathbf{E}}_n = Q_n, \quad \vec{\nabla} \cdot \vec{\mathbf{B}}_n = 0, \quad \partial \vec{\mathbf{E}}_n/\partial t = \vec{\nabla} \times \vec{\mathbf{B}}_n,$$
$$\partial \vec{\mathbf{B}}_n/\partial t = -\vec{\nabla} \times \vec{\mathbf{E}}_n.$$

We take a closer look at A_3 and A_4. Using the field conditions, we can write $\Omega_3(\kappa_2)$ as just time derivatives, using the field conditions:

$$\begin{aligned} \Omega_3(\kappa_2) &= \partial \mathbf{E}_1/\partial t + \vec{\nabla} \times \mathbf{B}_2 + \vec{\nabla} \times \mathbf{E}_1 + \partial \mathbf{B}_2/\partial t \\ &= \partial \mathbf{E}_1/\partial t + \partial \mathbf{E}_2/\partial t - \partial \mathbf{B}_1/\partial t + \partial \mathbf{B}_2/\partial t \\ &= \partial(\mathbf{E}_1 + \mathbf{E}_2)/\partial t + \partial(\mathbf{B}_2 - \mathbf{B}_1)/\partial t \end{aligned} \tag{103}$$

and $\Omega_4(\kappa_2)$ in the same way:

$$\begin{aligned} \Omega_4(\kappa_2) &= \partial \mathbf{E}_2/\partial t + \vec{\nabla} \times \mathbf{B}_1 + \vec{\nabla} \times \mathbf{E}_2 + \partial \mathbf{B}_1/\partial t \\ &= \partial \mathbf{E}_2/\partial t + \partial \mathbf{E}_1/\partial t - \partial \mathbf{B}_2/\partial t + \partial \mathbf{B}_1/\partial t \\ &= \partial(\mathbf{E}_1 + \mathbf{E}_2)/\partial t + \partial(\mathbf{B}_1 - \mathbf{B}_2)/\partial t. \end{aligned} \tag{104}$$

Then O_3 describes an accelerating moving charge Q_1 with its quadratically changing electric field and Ω_3 describes electromagnetic field derivatives changing with a second order of time. There are not yet any geometrical restrictions, so these minor fields exist independently of the chosen axes. Because the charges are moving with a second order of time, the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ are also changing with a second order of time, so $\partial \mathbf{E}_1/\partial t$ and $\partial \mathbf{E}_2/\partial t$ are the linear functions of time. This can be identified as electromagnetic waves, propagating with a constant velocity through vacuum.

Thus A_3 and A_4 can be identified as electrical attraction or repulsion of charged particles, emitting electromagnetic waves that propagate with the velocity of light through vacuum.

The charges can move toward or away from each other; there are no other possibilities. If they approach each other, there exists a point where they will meet; this would betray their position, which is contradictory to the independence of geometry. Hence the total charge must be zero, so for approaching charges, existing independent of geometry:

$$Q_1 = -Q_2 \tag{105}$$

is valid. We assume that for equal charges the remaining possibility is valid, so charges, existing independently of geometry, which are moving away from each other, are equal.

Next we add the set of *spacetime* appearances (41) to the set of *marked* appearances (101). The classical marking part in O_2 is in all cases restricted to a space, so only fields are observed, by which $\{0, \vec{\mathbf{O}}\}$ reduces to just the field part $\{\vec{\mathbf{O}}\}$; brackets are suppressed. Because the major points are generally not coinciding, $P_1 \cap S^2 \backslash P_2 = P_1$ and $P_2 \cap S^1 \backslash P_1 = P_2$. Only in the case of $P_1 = P_2$ do they reduce to empty sets. The major codomain $S^1 \backslash P_1 \cap S^2 \backslash P_2$ is generally written as $S^1 \backslash \{P_1, P_2\}$.

Then we obtain the general expression: A phenomenon, generated by two interacting Heisenberg events H_1 and H_2:

$$H_n\!:\!\{D_n,U^n,d_n,u^n\}=\left\{\begin{matrix}\{P_n,Q_n,\vec{\mathbf{E}}_n(t),t\}\\ \{S^n\setminus P_n,Q_n\times i,\vec{\mathbf{B}}_n(t),i\times t\}\\ \{p_n,\vec{\nabla},dt\}\\ \{s,\partial/\partial t,i\times dt\}\end{matrix}\right\}, \tag{106}$$

is described by the set of marked spacetime appearances:

$$A(\kappa_1,\kappa_2)=\left\{\begin{matrix}\{\{P_1\cap P_2,\{Q_1+Q_2,\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2\},t\},\{s^1\cap s^2\cap(P_1\bowtie s^2)\cap(P_2\bowtie s^1),\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t,t\}\}\\ \{\{S^1\setminus P_1\cap S^2\setminus P_2,\vec{\mathbf{O}},0\},\{[p_1\cap p_2],\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2),dt\}\}\\ \{\{P_1\cap S^2\setminus P_2,\{Q_1,\vec{\mathbf{E}}_1\},t^2\},\{[s^1\cap p_2\cap(P_1\bowtie p_2)],\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2),dt^2\}\}\\ \{\{P_2\cap S^1\setminus P_1,\{Q_2,\vec{\mathbf{E}}_2\},t^2\},\{[s^2\cap p_1\cap(P_2\bowtie p_1)],\partial(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1),dt^2\}\}\end{matrix}\right\}$$

$$\text{with } P_i\bowtie s^j=s^j(\text{if } P_i\in s^j),\quad P_i\bowtie s^j=\varnothing\ (\text{if } P_i\notin s^j),\quad P_i\bowtie p_j=p_j(\text{if } P_i\in p_j),\quad P_i\bowtie p_j=\varnothing\ (\text{if } P_i\notin p_j) \tag{107}$$

on the conditions

$$\vec{\nabla}\cdot\vec{\mathbf{E}}_n=Q_n;\vec{\nabla}\cdot\vec{\mathbf{B}}_n=0;\partial\vec{\mathbf{E}}_n/\partial t=\vec{\nabla}\times\vec{\mathbf{B}}_n;\partial\vec{\mathbf{B}}_n/\partial t=-\vec{\nabla}\times\vec{\mathbf{E}}_n.$$

A. Identification of the second quality

We consider the results of combining spacetime presence with charge by describing again the cases of Sec. III B with opposite or equal charges, or both. The addition of marking to the set of appearances is not simply a matter of providing more information; it may, for instance, restrict the case from an astronomic to an atomic level. The indicated field and charge items are only observable as far as the corresponding element supplies the necessary time dependence. If the described geometry in an element is empty, charges cannot be localized, so they are not observable; in that case, the corresponding vector fields exist independent of geometry, only restricted by accompanying attributes of time. If the described geometry in an element is not empty, charges and vector fields are restricted to this geometry as well as to the indicated attribute of time; the possibility of charges moving over an object is a new item.

The set for two qualities will at first be adapted to the set with one quality, as derived in Sec. III B. After that, the third and fourth elements will be considered at first; this is chosen to give priority to accelerated movements above movements of lower order.

General overview for each element of the set of marked spacetime appearances:

The first appearance A_1: In the classical part O_1, the marking set $\{Q_1+Q_2,\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2\}$ is restricted to $P_1\cap P_2$.

If $P_1=P_2$, then it reduces to just charge Q_1+Q_2, and if $P_1\cap P_2=\varnothing$. Then marking exists independently of geometry. The charges cannot be localized, so the set reduces to $\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2$.

The complementary part Ω_1 contains $\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t$ and the time attribute is t; thus $\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t$ is only observable in the minor geometry if it is not changing with time, in other words, if $\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2$ is constant or changing with the first order of time. Mind that, if the charges are accelerating, nothing is observable in this static element.

The second appearance A_2: In all cases, the classical part O_2 describes zero charge and zero fields. The complementary part Ω_2 describes the rotation of magnetic fields $\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2)$ restricted to $[p_1\cap p_2]$, which is a minor point $P(p_1\cap p_2)$ on the circle $p_1\cap p_2$, or a point of contact $X(p_1\cap p_2)$, or an empty set $\varnothing$. If a point is described, a magnetic vector is attached to it and no field exists in the remaining space. If the set is empty, the rotational magnetic field exists independently of geometry in the indicated space.

The third and fourth appearances A_3 and A_4: If $P_1\cap S^2\setminus P_2=P_1$, then $\{Q_1,\vec{\mathbf{E}}_1\}$ reduces to Q_1; if $P_1=P_2$ so $P_1\cap S^2\setminus P_2=\varnothing$, then $\{Q_1,\vec{\mathbf{E}}_1\}$ reduces to $\vec{\mathbf{E}}_1$; brackets are suppressed. The field derivatives are restricted to $[s^1\cap p_2\cap(P_1\bowtie p_2)]$, which is a point on a dot or an empty set; if a point is described, a field vector is attached to it and no field exists in the surrounding space. If the set is empty, the field exists independently of geometry in the complete space. In that case, the field of electromagnetic waves propagates with the velocity of light through vacuum.

A_3 and A_4 are generally the same, in mirrored sense; however, geometric descriptions may be nonsymmetrical, which can lead to a difference.

B. Eight cases of two H-events, interacting in spacetime and charge

Case 1, equal charges: $P_1=P_2=P$, $r_1=r_2$, $Q_1=Q_2=Q$, $\vec{\mathbf{E}}_1=\vec{\mathbf{E}}_2=\vec{\mathbf{E}}$, $\vec{\mathbf{B}}_1=\vec{\mathbf{B}}_2=\vec{\mathbf{B}}$. This set was geometrically identified as free particles, real as well as virtual. Adding κ_2 in case the H-events are identical in spacetime as well as marking (so linked genes are equal to the gene itself), according to Eqs. (41) and (96):

$$A(\kappa_1,\kappa_2)=\begin{Bmatrix}\{\{P,Q,t\},\{s,\partial\vec{\mathbf{E}}/\partial t,t\}\}\\ \{\{S\setminus P,\vec{\mathbf{O}},0\},\{P(p),\vec{\nabla}\times\vec{\mathbf{B}},dt\}\}\\ \{\{\varnothing,\vec{\mathbf{E}},t^2\},\{\varnothing,\partial\vec{\mathbf{E}}/\partial t,dt^2\}\}\\ \{\{\varnothing,\vec{\mathbf{E}},t^2\},\{\varnothing,\partial\vec{\mathbf{E}}/\partial t,dt^2\}\}\end{Bmatrix}. \quad (108)$$

Because of the geometrical conditions, the point charges are not moving, so the marked sets in O_3 and O_4, appearing with a second order of time, are not observable; they reduce to empty sets.

Thus in $\Omega_1(\kappa_2)$, $\Omega_3(\kappa_2)$, and $\Omega_4(\kappa_2)$: $\partial\vec{\mathbf{E}}/\partial t=\vec{\mathbf{O}}$; in Ω_2 using one of the field conditions:

$$\vec{\nabla}\times\vec{\mathbf{B}}=\partial\vec{\mathbf{E}}/\partial t=\vec{\mathbf{O}}. \quad (109)$$

The set reduces to:

$$A(\kappa_1,\kappa_2)=\begin{Bmatrix}\{\{P,Q,t\},\{s,\vec{\mathbf{O}},t\}\}\\ \{\{S\setminus P,\vec{\mathbf{O}},0\},\{P(p),\vec{\mathbf{O}},dt\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,\vec{\mathbf{O}},dt^2\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,\vec{\mathbf{O}},dt^2\}\}\end{Bmatrix}. \quad (110)$$

Summarizing, this set describes a charged real particle and an unmarked virtual particle.

Case 1, opposite charges: $P_1=P_2=P$, $r_1=r_2$, $Q_1=Q$, and $Q_2=-Q$. According to Eq. (107),

$$A(\kappa_1,\kappa_2)=\begin{Bmatrix}\{\{P,0,t\},\{s,\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t,t\}\}\\ \{\{S\setminus P,\vec{\mathbf{O}},0\},\{P_m(p),\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2),dt\}\}\\ \{\{\varnothing,\vec{\mathbf{E}}_1,t^2\},\{\varnothing,\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2),dt^2\}\}\\ \{\{\varnothing,\vec{\mathbf{E}}_2,t^2\},\{\varnothing,\partial(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1),dt^2\}\}\end{Bmatrix}. \quad (111)$$

Because of the geometrical conditions, the marked sets in O_3 and O_4 reduce again to empty sets and $\partial\vec{\mathbf{E}}_1/\partial t=\partial\vec{\mathbf{E}}_1/\partial t=\vec{\mathbf{O}}$.

In $\Omega_2(\kappa_2)$, using a field condition,

$$\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2)=\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t=\vec{\mathbf{O}}. \quad (112)$$

Then $\Omega_3(\kappa_2)$ and $\Omega_4(\kappa_2)$ reduce to magnetic field derivatives [see Eqs. (103) and (104)], with $\vec{\mathbf{B}}_2=-\vec{\mathbf{B}}_2=-\vec{\mathbf{B}}$,

$$\Omega_3(\kappa_2)=\partial(\vec{\mathbf{B}}_2-\vec{\mathbf{B}}_1)/\partial t=-2\times\partial\vec{\mathbf{B}}/\partial t,$$

$$\Omega_4(\kappa_2)=\partial(\vec{\mathbf{B}}_1-\vec{\mathbf{B}}_2)/\partial t=2\times\partial\vec{\mathbf{B}}/\partial t. \quad (113)$$

The set reduces to

$$A(\kappa_1,\kappa_2)=\begin{Bmatrix}\{\{P,0,t\},\{s,\vec{\mathbf{O}},t\}\}\\ \{\{S\setminus P,\vec{\mathbf{O}},0\},\{P_m(p),\vec{\mathbf{O}},dt\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,-2\times\partial\vec{\mathbf{B}}/\partial t,dt^2\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,2\times\partial\vec{\mathbf{B}}/\partial t,dt^2\}\}\end{Bmatrix}. \quad (114)$$

A_1 and A_2: *no descriptions of marking*.

A_3 and A_4: the magnetic field derivatives are observable with a second order of time, so the differences between magnetic fields $\vec{\mathbf{B}}_2-\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_1-\vec{\mathbf{B}}_2$ change linearly with time and, although changing, they are opposite in each point of spacetime. These appearances can be considered as the potential existence of two geometrically coinciding antiparticles, having opposite magnetic spins. Thus this set describes a free neutron, having the potential to create a pair of antiparticles.

Case 2: $0<P_{12}<r$. Both major points exist inside a nonspherical minor codomain.

Geometrically the codomain $s^1\cap s^2$ is static, so there can be no movement of the major points; thus the charges are also unmoving, so $\partial\vec{\mathbf{E}}_1/\partial t=\vec{\mathbf{O}}$ and $\partial\vec{\mathbf{E}}_2/\partial t=\vec{\mathbf{O}}$. Then, according to Eq. (107),

$$\Omega_1(\kappa_2)=\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t=\vec{\mathbf{O}},$$

$$\Omega_2(\kappa_2)=\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2)=\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t=\vec{\mathbf{O}}, \quad (115)$$

and $\Omega_3(\kappa_2)$ and $\Omega_4(\kappa_2)$ reduce according to Eqs. (103) and (104).

Thus the set can be reduced to

$$A(\kappa_1,\kappa_2)=\begin{Bmatrix}\{\{\varnothing,\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2,t\},\{s^1\cap s^2,\vec{\mathbf{O}},t\}\}\\ \{\{S\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{P_m(p_1\cap p_2),\vec{\mathbf{O}},dt\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,\partial(\vec{\mathbf{B}}_2-\vec{\mathbf{B}}_1)/\partial t,dt^2\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,\partial(\vec{\mathbf{B}}_1-\vec{\mathbf{B}}_2)/\partial t,dt^2\}\}\end{Bmatrix}. \quad (116)$$

In Ω_1, the field is zero, so no electrical activity is present inside the codomain, independent of charges.

Equal charges: $\vec{\mathbf{E}}_1=\vec{\mathbf{E}}_2=\vec{\mathbf{E}}$; $\vec{\mathbf{B}}_2=\vec{\mathbf{B}}_1$. The set reduces to

$$A(\kappa_1,\kappa_2)=\begin{Bmatrix}\{\{\varnothing,2\times\vec{\mathbf{E}},t\},\{s^1\cap s^2,\vec{\mathbf{O}},t\}\}\\ \{\{S\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{P_m(p_1\cap p_2),\vec{\mathbf{O}},dt\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,\vec{\mathbf{O}},dt^2\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,\vec{\mathbf{O}},dt^2\}\}\end{Bmatrix}. \quad (117)$$

This can be identified as two protons, existing in a nucleus without repulsing or attracting each other.

Opposite charges: $\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2=\vec{\mathbf{O}}$ so A_1 gives no information about marking.

Because $\vec{\mathbf{B}}_2=-\vec{\mathbf{B}}_1=-\vec{\mathbf{B}}$,

$$\partial(\vec{\mathbf{B}}_2-\vec{\mathbf{B}}_1)=-2\times\partial\vec{\mathbf{B}}/\partial t \quad \text{and} \quad \partial(\vec{\mathbf{B}}_1-\vec{\mathbf{B}}_2)=2\times\partial\vec{\mathbf{B}}/\partial t. \tag{118}$$

The set reduces to

$$A(\kappa_1,\kappa_2)=\left\{\begin{array}{l}\{\{\varnothing,\varnothing,t\},\{s^1\cap s^2,\vec{\mathbf{O}},t\}\}\\ \{\{S\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{P_m(p_1\cap p_2),\vec{\mathbf{O}},dt\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,-2\times\partial\vec{\mathbf{B}}/\partial t,dt^2\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,2\times\partial\vec{\mathbf{B}}/\partial t,dt^2\}\}\end{array}\right\}. \tag{119}$$

This can be identified as neutral solid matter. The existence of marking is only manifest as the changing magnetic fields in the complementary parts Ω_3 and Ω_4; this can only originate in a turning of the considered H-events around an axis through its major point P.

If this case perpetuates—if more and more H-events are involved—then, because charged H-events are not repulsing each other out of the discus, at an astronomic level they can accumulate until a *black hole* develops, from which nothing can escape. The surrounding pellicles are assumed to constitute a band of many almost concentric circles with minor points traveling through it. The radius of the dense band could be identified as the *Scharzschildradius*. Presumably this band interacts with newly arriving H-events as a kind of trap, by which they are absorbed; to investigate this, the set has to be extended to more H-events.

Case 3: $r<P_{12}<2\times r$. There is a minor codomain; the major points are outside of it and not on the pellicles. Geometrically this set was identified as H-events in a gas, having the freedom to move with a constant velocity with respect to each other. Adding κ_2, according to Eq. (107),

$$A(\kappa_1,\kappa_2)=\left\{\begin{array}{l}\{\{\varnothing,\{Q_1+Q_2,\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2\},t\},\{\varnothing,\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t,t\}\}\\ \{\{S\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{P_m(p_1\cap p_2),\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2),dt\}\}\\ \{\{\varnothing,Q_1,t^2\},\{\varnothing,\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2),dt^2\}\}\\ \{\{\varnothing,Q_2,t^2\},\{\varnothing,\partial(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1),dt^2\}\}\end{array}\right\}. \tag{120}$$

A_3 and A_4: geometrically, there is no accelerated movement of the major points, so the charges, attached to these points, cannot accelerate; thus they are not observed in O_3 and O_4 and will be replaced by empty sets.

A movement with the first order of time is still possible; then $\partial\vec{\mathbf{E}}_1/\partial t$ and $\partial\vec{\mathbf{E}}_2/\partial t$ are constant vector fields and $\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t=\vec{\mathbf{C}}$ (with $\vec{\mathbf{C}}$ a constant vector field).

Ω_3 and Ω_4 reduce to $\vec{\mathbf{C}}+\partial(\vec{\mathbf{B}}_2-\vec{\mathbf{B}}_1)/\partial t$ and $\vec{\mathbf{C}}+\partial(\vec{\mathbf{B}}_1-\vec{\mathbf{B}}_2)/\partial t$, respectively, [see Eqs. (103) and (104)]; a constant term is not observable in this appearance, so this can be replaced by a zero field.

A_2: a rotational magnetic field is restricted to the minor point $P_m(p_1\cap p_2)$, moving with a constant velocity over the minor track $p_1\cap p_2$, with a vector $\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2)$ attached to it.

Thus the set can be reduced to

$$A(\kappa_1,\kappa_2)=\left\{\begin{array}{l}\{\{\varnothing,\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2,t\},\{\varnothing,\vec{\mathbf{C}},t\}\}\\ \{\{S\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{P_m(p_1\cap p_2),\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2),dt\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,\partial(\vec{\mathbf{B}}_2-\vec{\mathbf{B}}_1)/\partial t,dt^2\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,\partial(\vec{\mathbf{B}}_1-\vec{\mathbf{B}}_2)/\partial t,dt^2\}\}\end{array}\right\}. \tag{121}$$

A_3 and A_4: Being independent of geometry, the electromagnetic fields reduce to the single gene $\partial E/\partial t$. Because only a linear change is allowed, this can only be a constant vector field, which is not observable together with the second order of time attribute; the fields can be replaced by zero fields. Then for charges moving in opposite directions: $\vec{\mathbf{C}}=\vec{\mathbf{O}}$, so $\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2)=\vec{\mathbf{O}}$ and the set reduces to

$$A(\kappa_1,\kappa_2)=\left\{\begin{array}{l}\{\{\varnothing,\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2,t\},\{\varnothing,\vec{\mathbf{O}},t\}\}\\ \{\{S\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{P_m(p_1\cap p_2),\vec{\mathbf{O}},dt\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,\vec{\mathbf{O}},dt^2\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,\vec{\mathbf{O}},dt^2\}\}\end{array}\right\}. \tag{122}$$

In A_1, a static field is observable if the charges are equal; no field is observable if they are opposite.

In A_2 the minor point $P_m(p_1 \cap p_2)$ travels over $p_1 \cap p_2$. For charges moving in one and the same direction, which is a current: $\vec{\mathbf{C}}$ is nonzero and the set reduces to

$$A(\kappa_1,\kappa_2)=\left\{\begin{matrix}\{\{\varnothing,\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2,t\},\{\varnothing,\vec{\mathbf{C}},t\}\}\\ \{\{S\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{P_m(p_1\cap p_2),\vec{\mathbf{C}},dt\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,\vec{\mathbf{O}},dt^2\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,\vec{\mathbf{O}},dt^2\}\}\end{matrix}\right\}. \tag{123}$$

In A_2, the minor point $P_m(p_1 \cap p_2)$ is observable, carrying a magnetic rotational vector, traveling along a circular track. This can be identified as a circular magnetic field line around a current.

Case 4: $r_1=r_2=r$, $P_{12}=r$. The major points are attached to each others' pellicles.

Geometrically this set was identified as gravity or dark matter, depending on the direction of the accelerated movement. Adding κ_2, according to Eq. (107),

$$A(\kappa_1,\kappa_2)=\left\{\begin{matrix}\{\{\varnothing,\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2,t\},\{\varnothing,\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t,t\}\}\\ \{\{S^1\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{P_m(p_1\cap p_2),\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2),dt\}\}\\ \{\{P_1,Q_1,t^2\},\{P_m(s^1\cap p_2),\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2),dt^2\}\}\\ \{\{P_2,Q_2,t^2\},\{P_m(s^2\cap p_1),\partial(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1),dt^2\}\}\end{matrix}\right\}. \tag{124}$$

Remember that for geometrical reasons: $P_m(p_1 \cap p_2)=P_m(s^1 \cap p_2)=P_m(s^2 \cap p_1)$.

A_4: Classically, an accelerated charge, approaching or divergent, is observed. Complementarily, a minor point $P_m(s^2 \cap p_1)$, circling over the border of the dot $s^2 \cap p_1$ with an electromagnetic field vector attached, changes with the first order of time; its velocity is the velocity of light.

A_3: Identical, in mirrored orientation; in this appearance the orientation of the traveling minor point is opposite to that in A_4.

A_2: The minor point $P_m(p_1 \cap p_2)$ is observed as traveling over the intersection of pellicles $p_1 \cap p_2$, a one-dimensional circular path, with a constant velocity and a rotational magnetic field vector attached.

A_1: The H-events are accelerating; there are no static observations, so the fields can be replaced by zero fields and the set can be written as

$$A(\kappa_1,\kappa_2)=\left\{\begin{matrix}\{\{\varnothing,\varnothing,t\},\{\varnothing,\vec{\mathbf{O}},t\}\}\\ \{\{S^1\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{P_m(p_1\cap p_2),\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2),dt\}\}\\ \{\{P_1,Q_1,t^2\},\{P_m(s^1\cap p_2),\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2),dt^2\}\}\\ \{\{P_2,Q_2,t^2\},\{P_m(s^2\cap p_1),\partial(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1),dt^2\}\}\end{matrix}\right\}. \tag{125}$$

The charges are taken in the geometrically produced gravitational or expanding movements, with electromagnetic vectors attached to the minor points. Because A_3 and A_4 have nonempty geometries, the charges can accelerate both while approaching and diverging, independent of their signs. Apparently a balance between gravity and electrical activity exists. This requires closer investigation than is possible in this paper.

Case 5: $P_{12} > 2\times r$. No minor attributes are shared.

Geometrically, the most stable set was obtained with accelerated divergent major points; this was identified as the expanding universe; for approaching H-events, the case led to the creation of two new H-events. Because of the acceleration, there are no static observations, so the fields in A_1 can be replaced by zero fields. Thus the set with κ_2 added, according to Eq. (107), can be reduced to

$$A(\kappa_1,\kappa_2)=\left\{\begin{matrix}\{\{\varnothing,\varnothing,t\},\{\varnothing,\vec{\mathbf{O}},t\}\}\\ \{\{S^1\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{\varnothing,\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2),dt\}\}\\ \{\{P_1,Q_1,t^2\},\{\varnothing,\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2),dt^2\}\}\\ \{\{P_2,Q_2,t^2\},\{\varnothing,\partial(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1),dt^2\}\}\end{matrix}\right\}. \tag{126}$$

The geometries in Ω_3 and Ω_4 are empty: no minor points, so no gravity.

A_3 and A_4: In O_3 and O_4 repulsing point charges and in Ω_3 and Ω_4 electromagnetic waves, traveling with the velocity of light through vacuum.

In summary, this set describes repulsing or attracting charges, producing electromagnetic waves in vacuum.

Case 6: $P_{12}=2\times r$. The pellicles touch each other in X_m.

This set was geometrically identified as the creation of a pair of H-events in vacuum at the moment that two accelerating H-events touch each others' pellicles; it describes only this point of time t. Adding κ_2, according to Eq. (107),

$$A(\kappa_1,\kappa_2)=\begin{Bmatrix}\{\{\emptyset,\{Q_1+Q_2,\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2\},t\},\{\emptyset,\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t,t\}\}\\ \{\{S\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{X_m(p_1\cap p_2),\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2),dt\}\}\\ \{\{P_1,Q_1,t^2\},\{\emptyset,\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2),dt^2\}\}\\ \{\{P_{21},Q_2,t^2\},\{\emptyset,\partial(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1),dt^2\}\}\end{Bmatrix}. \tag{127}$$

A_3 and A_4: Accelerating motion of the point charges, producing electromagnetic fields.

Opposite charges: $Q_1=Q$ and $Q_2=-Q$; $\vec{\mathbf{E}}_1=-\vec{\mathbf{E}}_2$, $\vec{\mathbf{B}}_1=-\vec{\mathbf{B}}_2$; thus, A_1: $\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2=\vec{\mathbf{O}}$ and $\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t=\vec{\mathbf{O}}$.

A_2: $\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2)=\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t=\vec{\mathbf{O}}$; this zero field is restricted to point X_m.

Because X_m is the coincidence of two pellicle points X_{m1} and X_{m2} and the charges are opposite, the zero vector can be considered as the sum of $\vec{\nabla}\times\vec{\mathbf{B}}_1$ attached to X_1 and $\vec{\nabla}\times\vec{\mathbf{B}}_2$ attached to X_2, with $\vec{\nabla}\times\vec{\mathbf{B}}_1+\vec{\nabla}\times\vec{\mathbf{B}}_2=\vec{\mathbf{O}}$. Therefore, the set can be written as

$$A(\kappa_1,\kappa_2)=\begin{Bmatrix}\{\{\emptyset,\emptyset,t\},\{\emptyset,\vec{\mathbf{O}},t\}\}\\ \{\{S\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{\{X_{m1},X_{m2}\},\vec{\mathbf{O}},dt\}\}\\ \{\{P_1,Q,t^2\},\{\emptyset,\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2),dt^2\}\}\\ \{\{P_2,-Q,t^2\},\{\emptyset,\partial(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1),dt^2\}\}\end{Bmatrix}. \tag{128}$$

Ω_2: The two points X_{m1} and X_{m2} are moving with a first order of time; they exist at one and the same place, so they can only be moving apart from each other with a constant velocity; all directions in the space $S\setminus\{P_1,P_2\}$ are allowed, which implies that movement in the direction of P_1 or P_2 is impossible.

In Ω_3 and Ω_4, two electromagnetic fields are described, which at point of time t are propagating independent of geometry with the velocity of light through vacuum. Thus these fields also exist in the pellicle points X_{m1} and X_{m2}. Because of compatibility of distinct set elements, the two minor points and the propagating fields exist simultaneously (although they cannot be *observed* simultaneously). Thus the constant velocity as described in Ω_2 is equal to the velocity of light and the minor points are the locations of two photons. In summary, this set describes the creation of a pair of photons with opposite magnetic spin, diverging from each other.

This phenomenon is also called action at a distance. The contact between the H-events, by which the photons continue to have opposite spins, is provided by the interaction of their major spaces $S^1\setminus P_1$ and $S^2\setminus P_2$, resulting in the space $S\setminus\{P_1,P_2\}$ in which the photons move. The occurrence of the phenomenon that two accelerated divergent photons with opposite spin stay in contact with each other, called the Einstein–Podolsky–Rosen–Bohm thought experiment, was proven by Aspect *et al.*[11]

Case 7: $P_{12}=r$ and $r_1 \gg r_2$. P_1 is attached to the large pellicle p_2.

Geometrically, the H-events are turning around each other; the complete dot, including P_2, moves over p_1 and is simultaneously turning around the axis P_{12}. According to Eq. (107),

$$A(\kappa_1,\kappa_2)=\begin{Bmatrix}\{\{\emptyset,\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2,t\},\{\emptyset,\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t,t\}\}\\ \{\{S\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{P_m(p_1\cap p_2),\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2),dt\}\}\\ \{\{\emptyset,Q_1,t^2\},\{\emptyset,\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2),dt^2\}\}\\ \{\{\emptyset,Q_2,t^2\},\{P_m(s^2\cap p_1),\partial(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1),dt^2\}\}\end{Bmatrix}. \tag{129}$$

For opposite charges, the field term in O_1 reduces to zero, independent of movement, so in A_1, $\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2=\vec{\mathbf{O}}$ and $\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t=\vec{\mathbf{O}}$; thus in A_2, $\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2)=\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t=\vec{\mathbf{O}}$.

A_3 and A_4: The charges cannot be observed because the major points are not observable; they will be replaced by zero.

Due to Eqs. (103) and (104), Ω_3 reduces to $\partial(\vec{\mathbf{B}}_2-\vec{\mathbf{B}}_1)/\partial t$, independent of geometry; Ω_4 reduces to $\partial(\vec{\mathbf{B}}_1-\vec{\mathbf{B}}_2)/\partial t$, restricted to $P_m(s^2\cap p_1)$. With opposite magnetic fields, these terms can be reduced to $-2\times\partial\vec{\mathbf{B}}/\partial t$ and $+2\times\partial\vec{\mathbf{B}}/\partial t$, respectively. Thus the set can be reduced to

$$A(\kappa_1,\kappa_2)=\left\{\begin{matrix}\{\{\varnothing,\varnothing,t\},\{\varnothing,\varnothing,t\}\}\\ \{\{S\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{P_m(p_1\cap p_2),\vec{\mathbf{O}},dt\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,-2\times\partial\vec{\mathbf{B}}/\partial t,dt^2\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{P_m(s^2\cap p_1),+2\times\partial\vec{\mathbf{B}}/\partial t,dt^2\}\}\end{matrix}\right\}. \quad (130)$$

Classically, the charges are not observable; complementarily, a circular track can be observed in A_2 and a particle in an atom in A_4. According to the attached vector in A_4, it has a magnetic spin up or down, depending on whether the minor point travels clockwise or counterclockwise over the minor track. This can be identified as an electron moving in a shell around a nonobservable, oppositely charged nucleus. In summary, the set describes a hydrogen atom.

Case 8: $P_{12}=r_1$ and $r_2\ll r_1$. The second pellicle is inside the first and P_1 is attached to it, so $P_1\in p_2$. This could be geometrically identified as a basis for a virtual particle as well as a hydrogen atom. According to (107):

$$A(\kappa_1,\kappa_2)=\left\{\begin{matrix}\{\{\varnothing,\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2,t\},\{\varnothing,\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2)/\partial t,t\}\}\\ \{\{S^1\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{\varnothing,\vec{\nabla}\times(\vec{\mathbf{B}}_1+\vec{\mathbf{B}}_2),dt\}\}\\ \{\{\varnothing,Q_1,t^2\},\{P_m(p_2),\partial(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_1+\vec{\mathbf{B}}_2),dt^2\}\}\\ \{\{\varnothing,Q_2,t^2\},\{\varnothing,\partial(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1)/\partial t+\vec{\nabla}\times(\vec{\mathbf{E}}_2+\vec{\mathbf{B}}_1),dt^2\}\}\end{matrix}\right\}. \quad (131)$$

A_3 and A_4: The charges cannot be observed because the major points are not observable; they will be replaced by zero.

If the charges are opposite, analogous to case 7, the set can be reduced to

$$A(\kappa_1,\kappa_2)=\left\{\begin{matrix}\{\{\varnothing,\varnothing,t\},\{\varnothing,\vec{\mathbf{O}},t\}\}\\ \{\{S^1\setminus\{P_1,P_2\},\vec{\mathbf{O}},0\},\{\varnothing,\vec{\mathbf{O}},dt\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{P_m(p_2),-2\times\partial\vec{\mathbf{B}}/\partial t,dt^2\}\}\\ \{\{\varnothing,\varnothing,t^2\},\{\varnothing,2\times\partial\vec{\mathbf{B}}/\partial t,dt^2\}\}\end{matrix}\right\}. \quad (132)$$

The phenomenom can only be complementarily observed in A_3; there is no circular track. The set describes a virtual particle, existing in the neighborhood of another, oppositely charged, nonobservable H-event.

V. CONCLUSION

By involving uncertainty in the measurement problem from scratch, it is possible to construct a complementary language that seems to be suitable to describe the physical universe fully. The chosen nomenclature, similar to that used in genetics, appears to be useful: a chromosome is a mathematical item, containing the information about how a phenomenon can manifest itself if it possesses the attributes of geometry, time, and marking. The observational descriptions cover a large variety of experimental results. In the observational descriptions, all four forces of nature can be recognized; it is possible to identify an electron, a neutron, a proton, dark matter, action at a distance, circular magnetic field lines, a magnetic spin vector, antiparticles, and virtual particles. Using the model, the Einstein–Podolsky–Rosen–Bohm paradox is solved and turned into a demonstration of the influence of minor attributes; the Cat Paradox is solved and turned into a Cat Uncertainty Amplifier.

The pellicle, which is constructed as a minor determinate geometric attribute, appears to be extremely useful, although its surface is even thinner than a point. On an atomic scale this object appears as quantum-mechanical rings, becoming clearer as the experiment progresses; on an astronomical scale it is observable by its simultaneous multitude. Experimental evidence is required for the predicted existence of circular pellicle tracks in the reversed situation of the gravitational lens, as observed by the Hubble space telescope.

Clearly, by giving uncertainty its proper conceptual place, a basis of a unification theory in the form of a powerful model is obtained. The new mathematical language I am advocating is much more convenient than standard languages for deriving the laws of Maxwell and offers a deeper understanding of all phenomena of nature in relation to each other.

ACKNOWLEDGMENTS

I want to express my gratitude to Wim Graef for his indefatigable support and Patrick de Saevsky for his essential comment.

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